Problem Set 5: Universal Law of Gravitation; Circular Planetary Orbits

Design Engineering Challenge: "The Big Dig" 2.007 Contest
Evaluation of Scoring Concepts: Spinner vs. Plower

PROMBLEM 1: Draw a free-body-diagram of a sphere in a hole, like is used to hold the shot-puts and the balls.

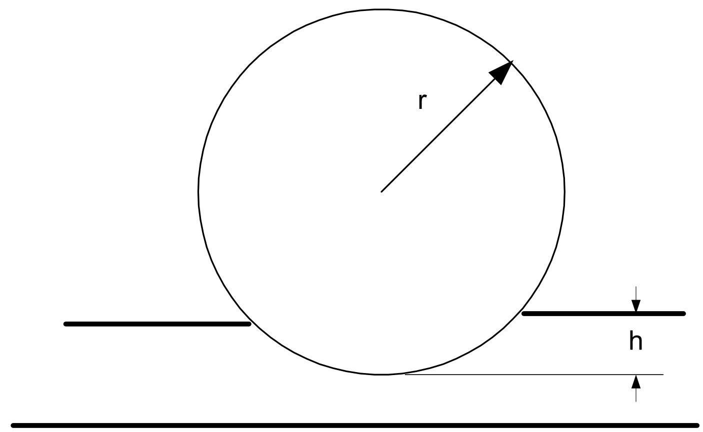


Figure 1: Diagram of a sphere in a hole

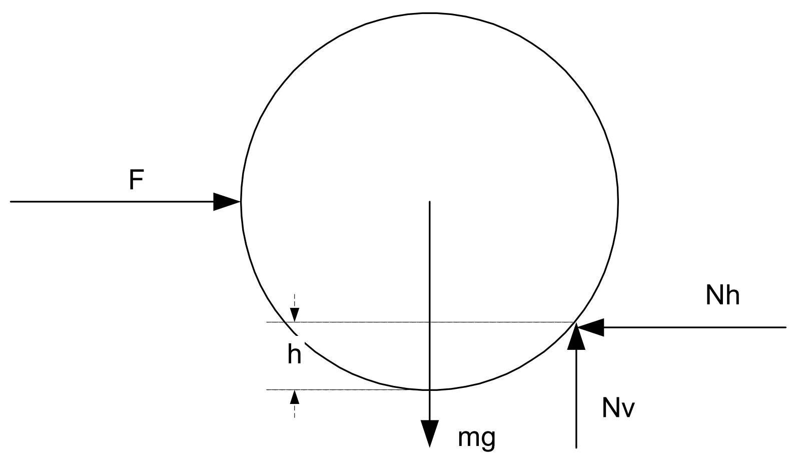


Figure 2: FBD of sphere in a hole.

Assume no friction over line contact.

PROMBLEM 2: The force condition for making a ball or shot-put to just rise up out of the hole.

Summing the forces in the x-direction first with right as the positive direction:

$$\underset{+}{\rightarrow} \sum F_x = 0 = F - N_h \quad \Rightarrow F = N_h \tag{1}$$

Summing the forces in the y-direction with up as the positive direction:

$$\uparrow_{+} \sum F_y = 0 = -mg + N_v \quad \Rightarrow N_v = mg \tag{2}$$

Lastly summing moments around the contact point:

$$\sum M_{contact\_point} = 0 = -F(r-h) + mg\sqrt{2rh-h^2} \quad \Rightarrow F = mg\frac{\sqrt{2rh-h^2}}{r-h} \tag{3}$$

Note for 0<h<r, the above moment arm for N_v was obtained from **Figure 3**, which results in equation (4).

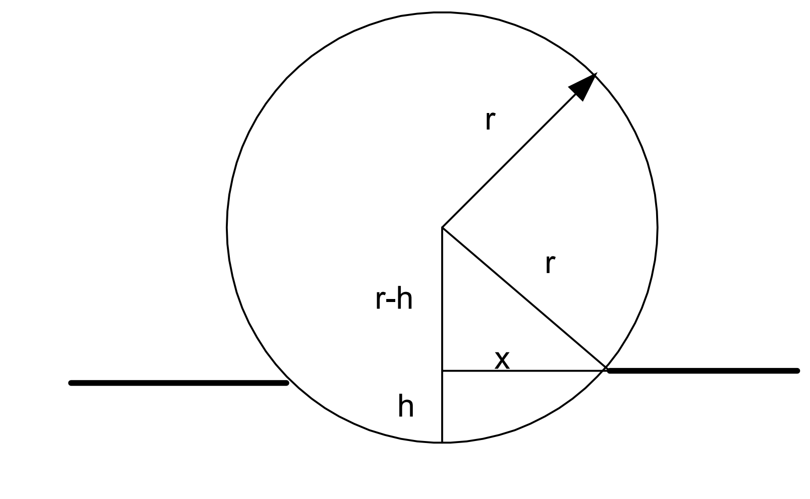


Figure 3: Dimensions of sphere.

$$x^2 + (r-h)^2 = r^2 \quad \Rightarrow x = \sqrt{2rh-h^2} \tag{4}$$

PROBLEM 3: The force condition for making a ball shot-put to leave the hole and start rolling across the platter.

This problem is just like problem 2 except there is an acceleration term. Summing the forces in the x-direction first with right as the positive direction:

$$\underset{+}{\rightarrow} \sum F_x = ma_x = F - N_h$$
$$=> F = ma_x + N_h \tag{5}$$

Summing the forces in the y-direction with up as the positive direction:

$$\uparrow_{+} \sum F_y = 0 = -mg + N_v$$
$$=> N_v = mg \tag{6}$$

Lastly summing moments around the contact point:

$$\sum M_{contact\_point} = 0 = -F(r-h) + mg\sqrt{2rh - h^2}$$
$$=> F = ma_x + mg\frac{\sqrt{2rh-h^2}}{r-h} \tag{7}$$

where the moment arm for N_v was obtained from **Figure 3**.

Note the difference between Equations (3) and (9) is the latter has an additional term of ma_x.

PROBLEM 4: Angular velocity of the platter must be achieved in order to meet the force conditions in (3)?

For force conditions in problem (3), we need the centrifugal force.

$$F = F_{centrifigual} = ma_{radial} = m\frac{v^2}{R} \tag{8}$$

but the linear velocity is equal to the angular velocity times the distance between the center of platter to center of ball which is denoted by R, refer to Figure 4. Thus equation (8) becomes:

$$F_{centrifigual} = m\frac{(\omega R)^2}{R} \tag{9}$$
$$=> F = m\omega^2 R$$

Substituting for F found in problem 3 into equation (9) produces:

$$ax + g\frac{\sqrt{2rh - h^2}}{r - h} = \omega^2 R \tag{10}$$
$$=> \omega = \sqrt{\frac{a_x}{R} + g\frac{\sqrt{2rh - h^2}}{R(r - h)}}$$

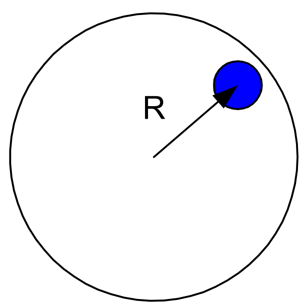


Figure 4: Distance between center of platter to center of shot-put.

PROBLEM 5: How hard would a "lasso" have to pull (or a blade to push) in order to meet the force conditions in (3)?

The amount of force required would be the same as equation (7):

$$F = ma_x + mg\frac{\sqrt{2rh - h^2}}{r - h} \tag{11}$$

PROBLEM 6: What is a better concept for liberating the shot-puts or hockey balls, spinning the platter or pulling or pushing them off?

Comparing equations (3), (7), and (11), the x direction acceleration needs to be considered for liberating the ball while pushing or pulling it off where when rotating, the moment of inertia needs to be known. Thus pushing the ball requires less force, but as the platter is spun with the proper angular velocity, more balls fly off for that amount of force.

PROBLEM 7: How do considerations of machine design complexity and feasibility affect the overall best concept?

Spinning the Platter:

a. Can the motors of the car handle the amount of torque necessary to get the angular velocity needed? If shot-put balls can not be spun off, instead could the hockey balls be? Then push the shot-put off the platter?
b. Which is quicker when removing the balls from the platter, spin or push/pull?

Pulling/pushing:

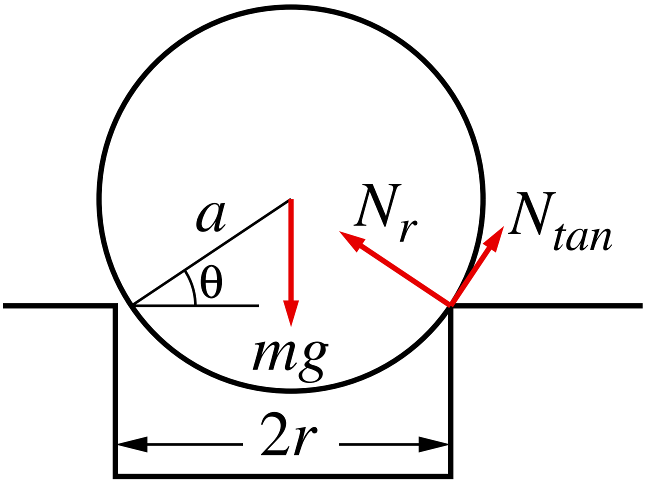
a. Can the motors handle pushing the shot-put off the platter plus the hockey balls in one effort or are multiple efforts needed?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01T — Fall Term 2003

Ball Coming Out of Socket

Here's how solve problem of a ball coming out of its socket on a rotating table without using non-intertial coordinate systems or knowing anything about moments. The figure below shows a ball just about to come out, in the sense that it makes contact with the table at only one point on the circumference of the socket. The ball has radius a and the socket has radius r.



The figure shows a free body diagram with all the forces acting on the ball shown in red. Newton's 2nd law for the vertical axis is

$$N_r \sin\theta + N_{tan} \cos\theta = mg$$

where

$$\theta = \cos^{-1}\left(\frac{r}{a}\right)$$

The horizontal force components can produce a maximum centipetal force

$$F_c = N_r \cos\theta - N_{tan} \sin\theta$$

If R is the radius of the ball's circular path, the ball starts to come out when

$$mR\omega^2 \geq mR\omega^2_{max} = N_r \cos\theta - N_{tan} \sin\theta$$

If there is no friction, $N_{tan} = 0$ and we obtain

$$\omega^2_{max} = \frac{g}{R\tan\theta}$$

The role of friction is discussed on the next page.

The role of friction can be understood if we think in a little more detail about how the ball escapes when there is no friction. As soon as $\omega > \omega_{max}$ the ball starts to slide up the edge of the socket. That increases θ, and therefore reduces the maximum centripetal force $N_r \cos\theta$. This is an unstable situation so the ball will escape. Because the ball slides up the edge of the socket as it escapes, any frictional force must act in the opposite direction to N_{tan} as drawn in the figure. Therefore

$$\begin{aligned}
N_r(\sin\theta - \mu\cos\theta) &= mg \\
R\omega_{max}^2 &= \frac{N_r(\cos\theta + \mu\sin\theta)}{m} \\
&= \frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}\, g \\
&= \frac{1 + \mu\tan\theta}{\tan\theta - \mu}\, g \\
\frac{d}{d\mu}(R\omega_{max}^2) &= \frac{1 + \tan^2\theta}{(\tan\theta - \mu)^2}\, g
\end{aligned}$$

So you can see that adding some friction makes it harder to get the ball out, in the sense that ω_{max} increases. That is consistent with our intuitive expectation.

However, there is another possibility: that the ball will roll rather than slide over the edge of the socket. In that case, the value for ω_c is the same as for the frictionless case. This means that only effect of friction is to make the ball roll out rather than slide out of the socket. It does not change the critical rotation speed.

If you are not sure you believe this, I found the easiest way to show it was to go into the rotating coordinate system and calculate the torques of the weight and of the centrifugal force about the contact point at the edge of the socket.