Problem Set 6: Static Equilibrium and Torque, Work-Kinetic Energy Theorem, Work Done by Friction and other Dissipative Forces

Design Engineering Challenge: "The Big Dig" 2.007 Contest
Hockey Puck Handling Strategies

***PROBLEM 1*:** The hockey pucks can be rolled into position if handled carefully, but if they tip over, they will have to be pushed. Draw the FBD of a hockey puck being pushed (slid on its face) and the FBD of a hockey puck being rolled on its edge.

Pushed:

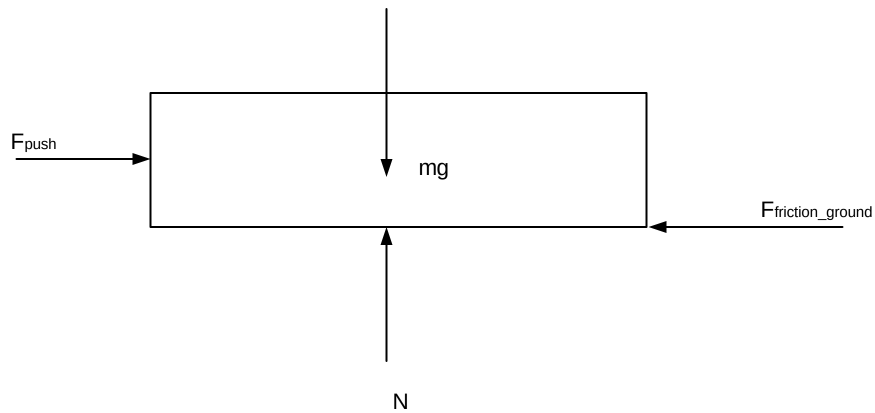

Figure 1: FBD of a hockey puck being pushed

Rolled:

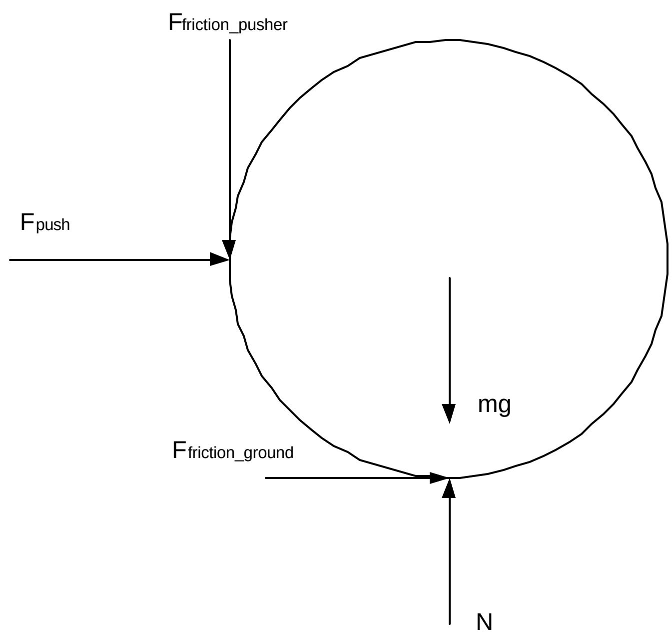

Figure 2: FBD of a hockey puck being rolled.

PROBLEM 2**:** How can you determine the coefficient of friction between the hockey puck and the ramp material?

Static friction:

Place the puck on a surface covered with ramp material and increase the angle between the ramp and ground (horizontal surface) until the puck slides with constant velocity.

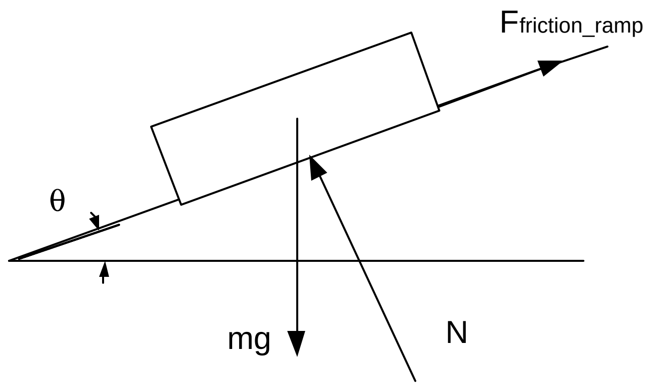

Figure 3: Hockey puck about to slide down an inclined plane.

$$\sum F_x = mg\sin q - F_{friction\_ramp} = 0 \quad \textbf{(1)}$$

$$F_{friction\_ramp} = \frac{mg\sin q}{m} \quad \textbf{(2)}$$

$$\sum F_y = N - mg\cos q = 0 \quad \textbf{(3)}$$

$$N = mg\cos q \quad \textbf{(4)}$$

But $F_{friction\_ramp} = \mu N$. Plugging this into equation (2) and substituting in for the normal force which is found in (4) produces:

$$\cos q = \frac{\sin q}{m} \quad \textbf{(5)}$$

$$=> m = \tan q$$

Once the puck starts to slide as the ramp's angle is increased, read this value for theta and plug into equation (5).

Another way is to connect a spring gauge to the end of a block of material you are trying to find the coefficient of friction for.

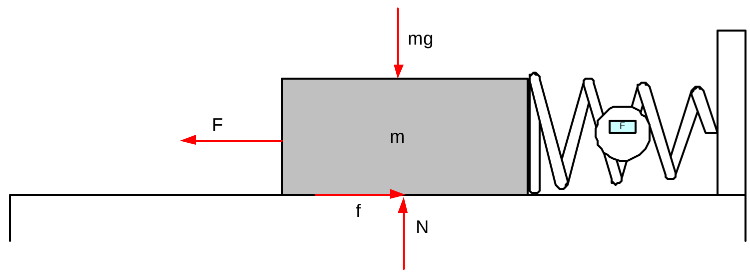

Figure 4: Spring gauge system to determine the coefficient of friction.

Summing the forces in the x and y directions produces a relationship between the coefficient of friction, weight, and the force applied.

$$\sum F_x = F - f \qquad \text{(6)}$$
$$=> F = mN$$

$$\sum F_y = N - mg \qquad \text{(7)}$$
$$=> N = mg$$

$$m = \frac{F}{mg} \qquad \text{(8)}$$

Once the block starts to move as a force is applied to it, read the value of the force off the spring gauge. This value can be plugged into equation (8) to find the coefficient of friction.

***PROBLEM 3*:** How much work does it take to slide a hockey puck up the ramp?

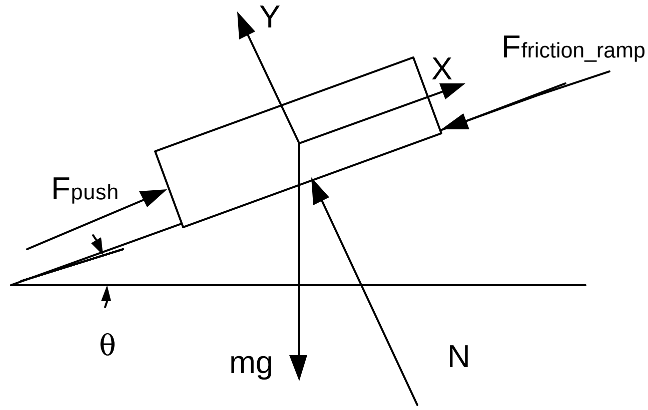

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Figure 5: Pushing a hockey puck up an inclined plane.
Only forces in the x-direction are considered.

Work = Force X distance.

$$\sum F_x = F_{Push} - mg\sin\theta - F_{friction\_ramp} \tag{9}$$

From the y direction the Normal force, N is found to me

$$N = mg\cos\theta \tag{10}$$

Thus the forces in the x-direction are

$$\begin{aligned} \sum F_x &= F_{Push} - mg\sin\theta - \mu mg\cos\theta \\ &=> F_{Push} - mg\left(\sin\theta + \mu\cos\theta\right) \end{aligned} \tag{11}$$

The total distance to travel is determined from Figure 6.

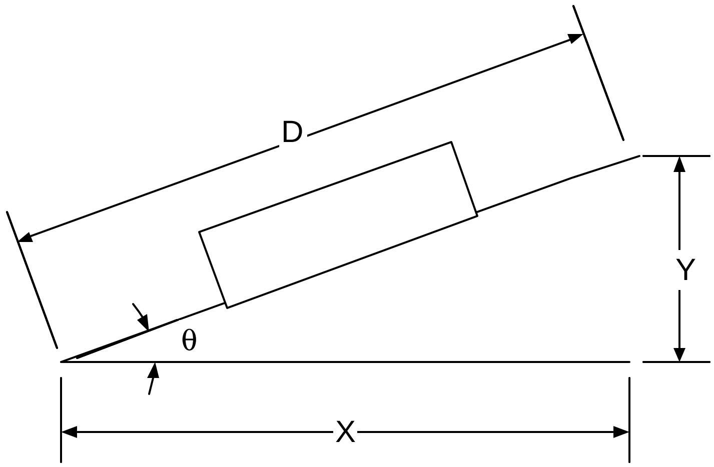

Figure 6: Variables of ramp.

Where X and θ are given dimensions on the table (reference 2.007 web site). This makes the value of D as

$$D = \frac{X}{\cos\theta} \tag{12}$$

Substituting this back into the work equation produces the final result of the amount of work done pushing a puck up the ramp.

$$W = \left(F_{Push} - mg\left(\sin\theta + \mu\cos\theta\right)\right)\frac{X}{\cos\theta} \tag{13}$$
$$\Rightarrow F_{Push}X - mg\left(\tan\theta + \mu\right)X$$

***PROBLEM 4*:** How much work does it take to roll a hockey puck up the ramp?

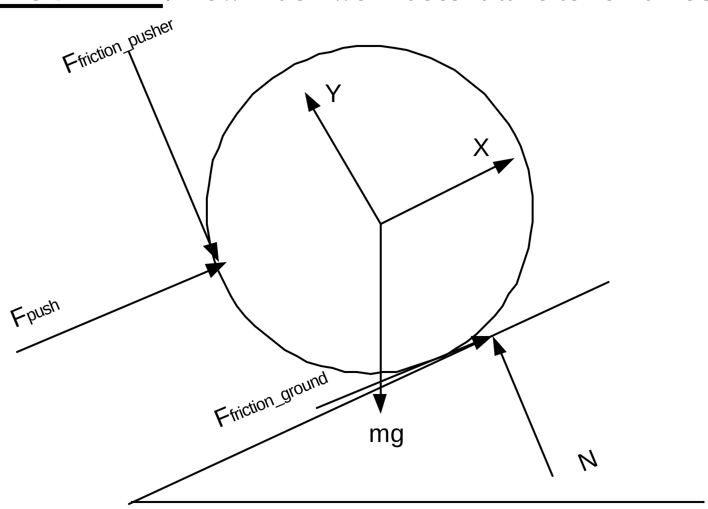

Figure 7: FBD of a hockey puck being rolled up an incline.

$$\sum F_x = F_{Push} - mg\sin q + F_{friction\_ramp} \tag{14}$$

From the y direction the Normal force, N is found to me

$$N = mg\cos q + F_{friction\_push} \tag{15}$$

Where $F_{friction\_push} = \mu_{Push}F_{Push}$. Substituting this value for the one in (14), the normal force from the ramp becomes:

$$N = mg\cos q + m_{Push}F_{Push} \tag{16}$$

The forces in the x-direction are:

$$\begin{aligned} &\sum F_x = F_{Push} - mg\sin q - m_{ramp}\left(mg\cos q + m_{Push}F_{Push}\right) \\ &=> F_{Push}\left(1 - m_{ramp}m_{Push}\right) - mg\left(\sin q + m_{ramp}\cos q\right) \end{aligned} \tag{17}$$

Substituting this back into the work equation produces the final result for the amount of work done rolling a puck up the ramp.

$$\begin{aligned} &W = \left(F_{Push}\left(1 - m_{ramp}m_{Push}\right) - mg\left(\sin q + m_{ramp}\cos q\right)\right)\frac{X}{\cos q} \\ &=> \frac{F_{Push}X}{\cos q}\left(1 - m_{ramp}m_{Push}\right) - mg\left(\tan q + m_{ramp}\right)X \end{aligned} \tag{18}$$

PROBLEM 5**:** Does it really matter, given the work required to increase the hockey puck's potential energy, whether you slide or roll the puck up the ramp?

Potential energy in this situation is equal to

$$U = mgh$$

The value of h is found from Figure 6, which is

$$h = X\tan q$$

Thus the potential energy is

$$U = mgX\tan q$$

The potential energy is the same for both cases, but the work is different because one hockey puck is rolling, less work, and the other is sliding, more work. Refer to equations (13) and (18).

***<u>PROBLEM 6</u>*:** How else might you get the hockey puck (s!) up the ramp quickly, and what would be the forces and powers involve?!

a) Carry the pucks on the machine and travel up the ramp. This removes the friction force due to the puck rolling against the car, but the additive force is the friction between the wheels and ground and the puck remaining on the back of the car. This is like problem set 3.

$$W = \left(F - m_{car} N\right) d$$
$$=> \left(F - m_{car}\left(m_{car} + m_{puck(s)}\right) g \cos q\right) d \qquad \textbf{(19)}$$

Power is work divided by time. Thus guestimating the amount of time it would take to drive the car up the ramp will give us the amount of power.

$$Power = \frac{Work}{time} \qquad \textbf{(20)}$$

b) Catapult/launch projectile.

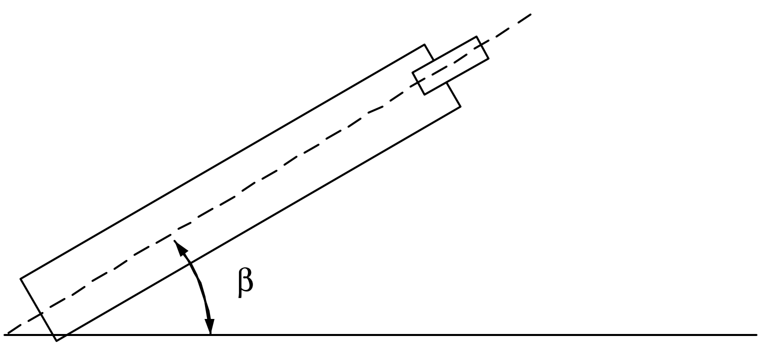

Figure 8: Launch hockey pucks instead of push or roll them.

$$F = \frac{m}{t}\sqrt{\frac{gD}{\sin 2b}} \qquad \textbf{(21)}$$

D is the length of the ramp as in Figure 6. Work is the force times the total distance the projectile is shot, which in our case the distance is X in Figure 6. This produces

$$W = \frac{m}{t}\sqrt{\frac{gD}{\sin 2b}}X \tag{22}$$

Power is also equal to the force times the velocity. In our case the velocity is the change in distance divided by time, which produces:

$$Power = \frac{m}{t}\sqrt{\frac{gD}{\sin 2b}}\,\frac{X}{time} \tag{23}$$

***PROBLEM 7*:** Guestimate what you think is a reasonable time for the motion of the hockey puck, given the total time for the contest, and now estimate what is the power required for the different strategies.

The Equation of power is given in (20). Guessing it will take a time T to travel up the ramp, this produces the following power for sliding and rolling the hockey puck:

Sliding:

$$Power = \frac{F_{Push}X - mg\left(\tan q + m\right)X}{T} \tag{24}$$

Rolling:

$$Power = \frac{F_{Push}X}{T\cos q}\left(1 - m_{ramp}m_{Push}\right) - mg\left(\tan q + m_{ramp}\right)\frac{X}{T} \tag{25}$$

Note, the student may pick any value for T given that the amount of time considers the time it takes it gets to the ramp and the total length of the contest of 45 seconds.

***PROBLEM 8*:** Remember the platter with the balls and shot-put? How much energy must be expended to get it spinning to the point where the hockey balls fly off? How much energy must be expended to get the shot-puts to fly off?

From Problem Set 5, we found

$$w = \sqrt{\frac{a_x}{R} + g\frac{\sqrt{2rh - h^2}}{R(r-h)}} \tag{26}$$

We also know the velocity of the platter is ωR_{eff} and the kinetic energy (KE) is

$$KE = \frac{1}{2}mv^2$$

$$=> \frac{1}{2}mR_{eff}{}^2 w^2 \qquad \textbf{(27)}$$

Where R_{eff} is the effective radius of the disk. Its value represents the position where most of the mass resides. This value is $R_{eff} = \frac{R}{\sqrt{2}}$.

This produces

$$KE = \frac{1}{2}mR_{eff}{}^2 \left(\frac{a_x}{R_{eff}} + g\frac{\sqrt{2rh - h^2}}{R_{eff}(r-h)} \right) \qquad \textbf{(28)}$$

Simplifying the above equation more, results in

$$KE = \frac{1}{2}mR_{eff} \left(a_x + g\frac{\sqrt{2rh - h^2}}{(r-h)} \right) \qquad \textbf{(29)}$$

***PROBLEM 9*:** How does this compare to the energy to move a puck up the ramp, and does this give you any feeling for what is a more efficient way to score?

Forces required to liberate hockey balls are small. Once the wheel is rotating if there is enough force to liberate shot-puts, go for it. If not, attempt to roll the hockey pucks. If the hockey pucks happen to fall/tip over onto a flat slide, push/slide them like a mad man/woman, but as the calculation shows, it is less work to roll the puck than slide it.