# Problem Set 9: Momentum and Collision Theory, Rigid Bodies, Kinematics 

## Design Engineering Challenge: "The Big Dig" 2.007 Contest Ball Pyramid breaking Concepts

## PROBLEM 1:



Figure 1: Projectile Mechanism
Not retractable. One shot, rapid deployment, little room for error, misalignment.
Would be better if could rapidly retract, but is not feasible. Far from target.


Figure 2: Telescope Mechanism
Retractable, multiple attempt fairly rapid deployment, room for error/ misalignment.

Can rapidly retract but not so important. Direct contact with target.

## PROBLEM 2:

Projectile:

$V_{o x}=V_{o} \cos \theta$
$V_{o y}=V_{o} \sin \theta$
At highest point,
$V_{y}=V_{o} \sin \theta-g t=0$
$t=\frac{V_{o} \sin \theta}{g}$
at R , total time $\mathrm{T}=2 \mathrm{t}$ where $\mathrm{T}=$ time to target
$R=\left(V_{o} \cos \theta\right) T=\frac{2 V_{o}^{2}(\sin \theta \cos \theta)}{g}=\frac{V_{o}^{2}(2 \sin \theta \cos \theta)}{g}=\frac{V_{o}^{2} \sin 2 \theta}{g}$

## Telescope:

Rotational: $T=\frac{2 \pi}{\omega} \quad C=2 \pi R \quad$ where, $?=\mathrm{rpm}$ of motor and $\mathrm{R}=$ radius of wheel.

Translational: $V=\omega R \quad t=\frac{L}{V} \Rightarrow t=\frac{L}{\omega R}$ prong extension

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T=\frac{D}{\omega R} \text { time to target }
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## PROBLEM 3:

$$
\begin{aligned}
& K E=\frac{1}{2} m V^{2} \\
& P E=M g h
\end{aligned}
$$



M: mass of stack
h : height of center of mass of stack
V : velocity of car
To fully pierce,
Conservation of energy (negligible losses)
$\frac{1}{2} m V^{2}=M g h \Rightarrow V=\sqrt{\frac{2 M g h}{m}}$
Conservation of momentum (perfectly elastic)
$m V_{c a r}=m V_{\text {stack }} \Rightarrow V_{\text {car }}=\frac{M}{m} V_{\text {stack }}$

## PROBLEM 4:

An experiment can be observed to verify conversation of energy approach. Run car at calculated velocity and see if stack is fully pierced.

