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Electrical and Information Data Book

1995 Version

Cambridge University Engineering Department

ELECTRICAL AND INFORMATION DATA BOOK

1994 Version
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Contents

1 GENERAL PHYSICAL CONSTANTS

Electron rest mass	m_e	9.109×10^{-31}kg
Proton rest mass	m_p	1.673×10^{-27} kg
Neutron rest mass	m_n	1.675×10^{-27} kg
Proton/Electron mass ratio	m_p/m_e	1.836×10^{3}
Electronic charge	e	-1.602×10^{-19} C
Electronic charge/mass ratio	e/m_e	1.759×10^{11} C/kg
Velocity of light in vacuo	c	2.998×10^{8} m/s
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ H/m
Permittivity of free space	ϵ_o	8.854×10^{-12} F/m
Planck Constant	h	6.626×10^{-34} J s
Boltzmann Constant	k	1.381×10^{-23} J/K
Stefan-Boltzmann Constant	σ	5.670×10^{-8} J/K^{4}m^{2}s
Molar number (Avogadro's constant)	N_A	6.022×10^{26} kmol^{-1}
Faraday Constant	F	9.649×10^{7} C/kmol
Standard Volume of Perfect Gas	V_o	22.41 m^3/kmol
Molar (universal) gas constant	$\bar{R}$	8.314×10^{3} J/K kmol
Ice point	T_o	273.15 K
Standard Atmospheric Pressure	P_o	1.01325×10^{5} N/m^2 (=1 atm) (10^5 N/m^2 = 1 bar)
Standard Acceleration	g	9.80665 m/s^2

2 PROPERTIES OF MATERIALS AND SOLID STATE PHYSICS (Typical values)

2.1 Metals and Alloys etc.

	Resistivity at 20 °C Ωm	Temp. Coeff. of Resistance K^{-1} at 20 °C	Temp. Coeff. of Expansion K^{-1}	Specific Heat-Capacity J/kg K	Thermal Conductivity W/m K	Melting Point °C
Copper	1.72×10^{-8}	39×10^{-4}	25.5×10^{-6}	380	385	1083
Aluminium	2.8×10^{-8}	40×10^{-4}	16.7×10^{-6}	880	200	660
Tungsten	5.5×10^{-8}	45×10^{-4}	4.4×10^{-6}	140	160	3370
Manganin	44.5×10^{-8}	0.1×10^{-4}	18×10^{-6}		26	910
Nichrome	103×10^{-8}	1.5×10^{-4}	17×10^{-6}	450	13	1350
Carbon	4500×10^{-8}	-5×10^{-4}	5.4×10^{-6}	840	1.7	3500
Iron	100×10^{-8}	54×10^{-4}	11.6×10^{-6}	250	67	1537
Stainless steel	72×10^{-8}	-	9×10^{-6}	500	16	1427

2.2 Dielectrics.

	Relative Permittivity	Dielectric Strength MV/m	tan δ at 50 Hz	tan δ at 1 MHz	tan δ at 1 GHz	Resistivity Ω m
Mica	6	200	25×10^{-4}	3×10^{-4}	3×10^{-4}	$10^{11} - 10^{15}$
Glass	5	20	6×10^{-4}	8×10^{-4}	12×10^{-4}	$10^{9} - 10^{12}$
Porcelain	6	30	220×10^{-4}	75×10^{-4}	100×10^{-4}	-
Polystyrene	2.5	20	0.5×10^{-4}	0.7×10^{-4}	3.3×10^{-4}	
P.T.F.E.	2.1	20	5×10^{-4}	2×10^{-4}	2×10^{-4}	$10^{15} - 10^{19}$
Transfr. Oil	2.2	15	4×10^{-4}	5×10^{-4}	30×10^{-4}	-
Alumina	8.5	-	20×10^{-4}	-	-	-
Quartz	3.8	20	10×10^{-4}	-	-	10^{16}
Polythene	2.3	20	2×10^{-4}	-	-	$10^{8} - 10^{14}$
Polycarbonates	3.1	-	50×10^{-4}	-	-	$10^{11} - 10^{14}$

2.3 Semiconductors (properties at 300 K).

	Energy Gap eV	Mobilities Electron m^2/Vs	Mobilities Hole m^2/Vs	Relative Permittivity
Germanium	0.67	0.39	0.19	16
Silicon	1.12	0.16	0.05	12
Gallium Arsenide	1.40	0.9	0.04	12.5
Indium Antimonide	0.16	7.0	0.07	17

2.4 Superconductors:

	T_c K	B_c or B_{c2} at 0 K tesla (T)
Al	1.2	0.01
Pb	7.2	0.08
Nb	9.2	0.08
Nb Sn	18.4	24
Y Ba Cu0	93	$\sim$ 100
T1 Ba Ca Cu0	125	$\sim$ 120

Critical current density is very variable:

Nb Sn will carry 10^9 A/m^2 in a field of 5 T at 4.2 K in wire form.

Y Ba Cu0 will carry 10^{10} A/m^2 in zero magnetic field at 77 K in thin film form.

Flux quantum $\frac{h}{2e} = 2.07 \times 10^{-15}$ J s C^{-1}.

Energy gap $\sim$ 3500 T_c.

2.5 Solid state physics for crystalline materials.

Density of states for nearly free electrons:

$$g(E) = \frac{4\pi(2m^*)^{3/2}}{h^3} E^{\frac{1}{2}}$$

where E is the energy measured from the bottom of the band.

For a semiconductor where $E_c - E_f > 3kT$, the electron density in the conduction band may be written as

$$n = N_c \exp\left\{-\frac{(E_c - E_f)}{kT}\right\}$$

where E_c is the energy of the bottom of the band and the effective density of states, N_c, is given by

$$N_c = 2\frac{(2\pi m^* kT)^{3/2}}{h^3} m^{-3}.$$

The equation of continuity for excess minority electrons is

$$\frac{\partial n}{\partial t} = -\frac{n}{\tau} + D\nabla^2 n + \mu\nabla.(n\underline{E}).$$

τ is the carrier life-time.
D is the carrier diffusion coefficient.
μ is the carrier mobility.
$\underline{E}$ is the electric field.

Einstein's relation between mobility and diffusion coefficient:

$$D = \frac{kT}{e}\mu.$$

2.6 Magnetic Materials

The number of magnetic materials in common use is very large and the data given here do not attempt to cover more than a few representative examples. For most purposes the materials may be grouped into four categories, as shown below, though there are some useful materials which lie outside these groups.

2.6.1 Group I: Materials used in the electrical power industry

These materials, notably cast steel and various silicon-iron alloys, are relatively inexpensive and have high values of maximum flux density. The silicon-iron alloys have moderately high permeabilities and fairly low hysteresis losses.

Magnetization curves are given in Fig. 1a.

2.6.2 Group II: The nickel-iron alloys

By comparison with Group I, these alloys are much more expensive, have much higher permeabilities, lower hysteresis loss and lower values of maximum flux density. They are widely used in the light-electrical industry. Several different alloys are available to provide different compromises between cost, permeability, maximum flux density and resistivity (to reduce eddy-current loss).

Small quantities of elements other than nickel and iron are sometimes added. Representative types of alloy are:-

Percentage of nickel	Trade Names	Initial relative permeability	Maximum Flux Density Wb/m^2	Resistivity Ωm
70-90	Mumetal Permalloy C etc.	10,000 to 30,000	0.8	6.0×10^{-7}
45-50	Radiometal Permalloy B etc.	1,800 to 2,400	1.6	5.5×10^{-7}
35-45	Rhometal Permalloy D etc.	1,500 to 2,000	1.3	9.0×10^{-7}

Magnetization Curves are given in Fig. 1b.

2.6.3 Group III: Permanent magnet materials

Several alloys of nickel and aluminium, with or without cobalt, copper and titanium, have been developed for the manufacture of permanent magnets. These materials have high remanent magnetism and high coercive force.

Magnets sintered from Barium Ferrite powder have been developed, and are characterised by low remanence and high coercive force.

Cobalt/rare earth materials which exhibit an extremely high maximum energy product are recent developments. Cobalt/samarium is available in two forms:

a.	Sintered metal:	Remanent flux density	0.87 T
		Coercive force	1280×10^3A/m
b.	Moulded powder in Epoxy:	Remanent flux density	0.435 T
		Coercive force	640×10^3A/m.

This material may be machined.

The demagnetization characteristic is linear between the remanent flux density (H=0) and the coercive force (B=0)

Representative demagnetization curves are shown in Fig. 2.

2.6.4 Group IV: Ferrites

In addition to the alloys mentioned above, a wide range of non-metallic ferrites has been developed for magnetic purposes. In general, the purely magnetic properties of the ferrites are inferior to those of the alloys, but the former have much higher resistivities and so can be used at much higher frequencies without serious eddy-current loss. A great many mixed ferrites of divalent metals have been prepared, but only three will be mentioned.

Manganese Zinc Ferrites.

Different materials in this range, with different proportions of manganese and zinc, have useful magnetic properties in the frequency range from 1 kHz to 20 MHz. Other properties lie in the ranges

Initial relative permeability	850-1500
Maximum flux density	0.34 - 0.40 Wb/m^2
Resistivity	0.5 - 1.0 Ωm.

Nickel Zinc Ferrites.

These materials cover a useful frequency range from 1 kHz to 200 MHz and have the following properties

Initial relative permeability	20 - 650
Maximum flux density	0.19 - 0.32 Wb/m^2
Resistivity	10^3 Ωm.

Barium Ferrite.

This material is quite different from those mentioned above and is used in the manufacture of permanent magnets. Representative properties are

Remanent flux density	0.36 Wb/m^2
Coercive force	1.1×10^5 A/m.

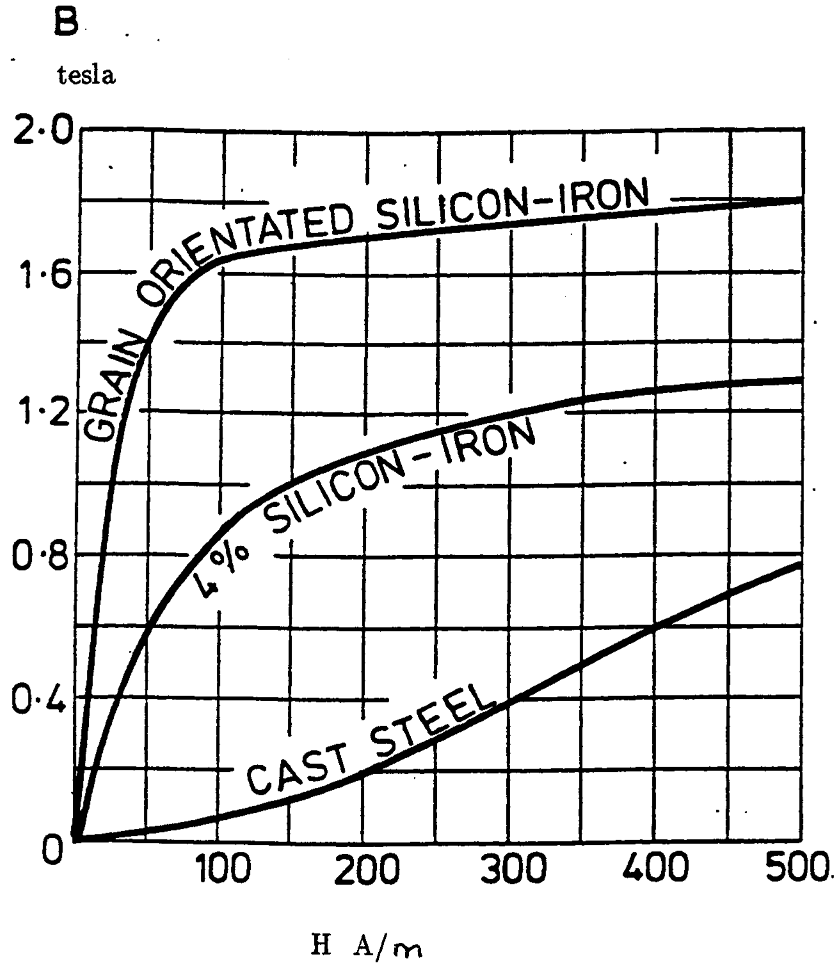

B
tesla
2·0
1·6
1·2
0·8
0·4
0
100
200
300
400
500
GRAIN ORIENTATED SILICON-IRON
4% SILICON-IRON
CAST STEEL
H A/m

Fig. 1a

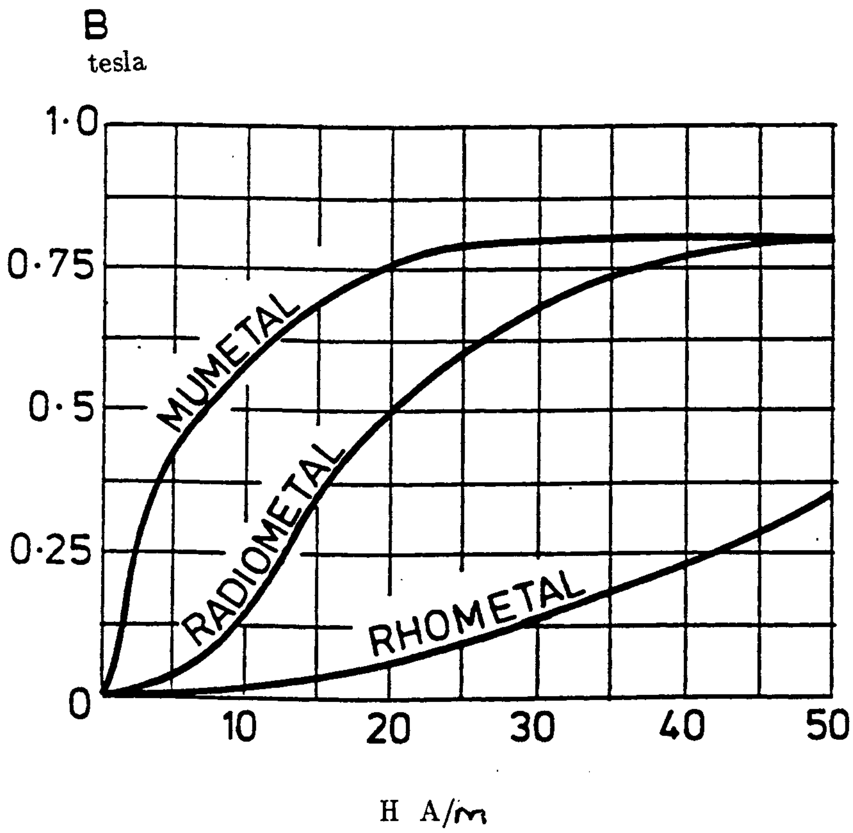

B
tesla
1·0
0·75
0·5
0·25
0
10
20
30
40
50
MUMETAL
RADIOMETAL
RHOMETAL
H A/m

Fig.1b

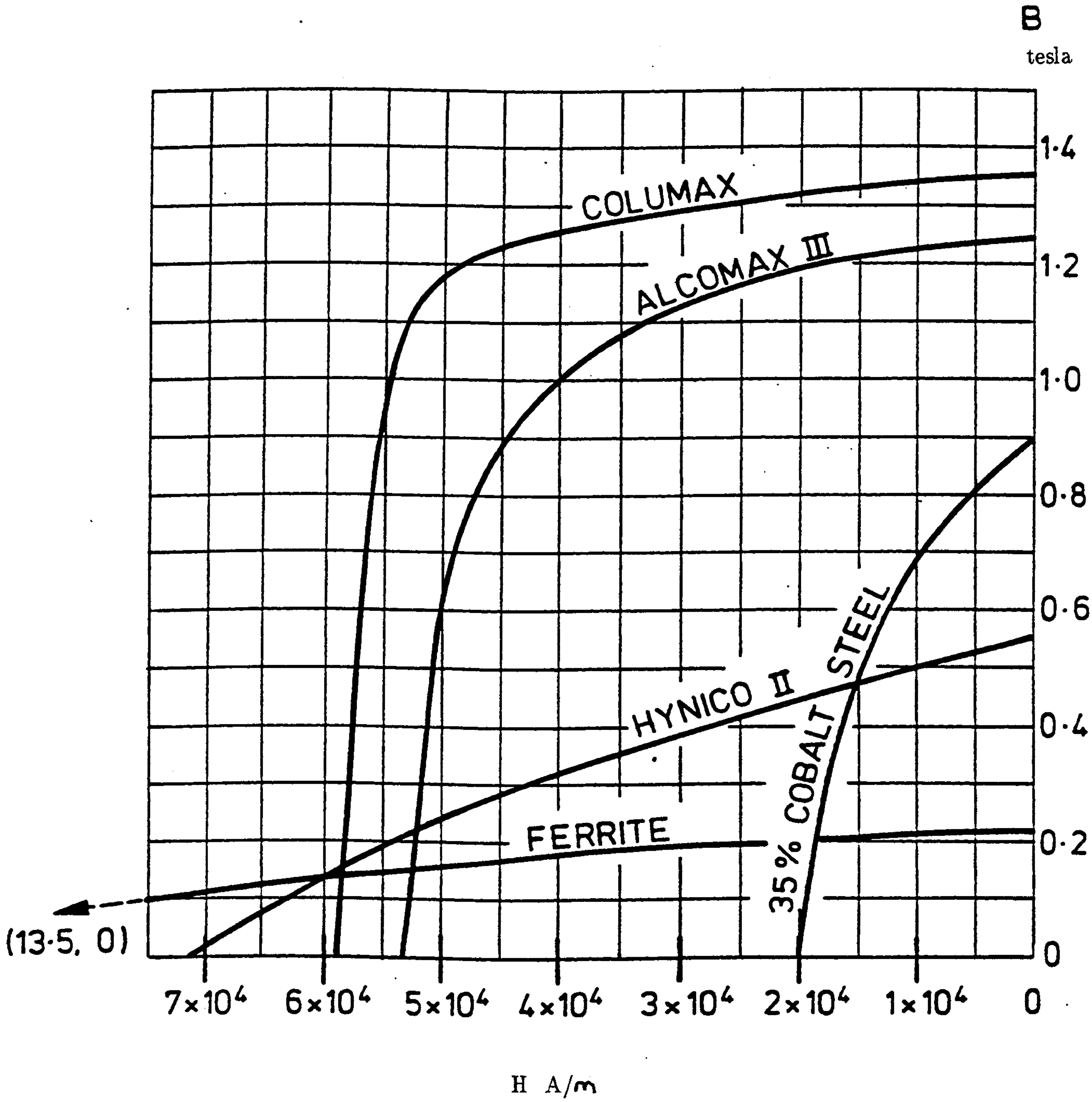

B
tesla
COLUMAX
ALCOMAX III
35% COBALT STEEL
HYNICO II
FERRITE
(13·5, 0)
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
7×10⁴
6×10⁴
5×10⁴
4×10⁴
3×10⁴
2×10⁴
1×10⁴
0
H A/m

Fig. 2

3 ELECTROMAGNETISM.

3.1 Electromagnetic Fields. Fundamental Variables and Equations.

I	:	Current, ampere (A), defined via force between parallel straight wires carrying current.
V	:	Potential, volt (V), defined such that 1 ampere × 1 volt = 1 watt.
Q	:	Charge, coulomb (C), defined as 1 ampere-second.
ρ	:	Charge density, C/m^3.
$\underline{J}$	:	Current density, A/m^2.
$\underline{B}$	:	Magnetic flux density, tesla (T), defined via force on a test current element.
$\underline{H}$	:	Magnetic field intensity, defined via the circuital law, A/m.
$\underline{P}$	:	Electrical polarisation, dipole density per unit volume, C/m^2.
$\underline{M}$	:	Magnetisation, A/m.
$\underline{E}$	:	Electric field intensity, defined via force on a test charge, V/m.
$\underline{D}$	:	Electric flux density, defined via Gauss's law, C/m^2.
σ	:	Electrical conductivity, siemen/m.
ε	:	Dielectric permittivity, F/m. (For free space $\varepsilon_0 = 8.854\ldots \times 10^{-12}$ F/m.)
ε_r	:	Relative permittivity, $\varepsilon_r = \varepsilon/\varepsilon_0$.
μ	:	Magnetic permeability, H/m. (For free space $\mu_0 = 4\pi \times 10^{-7}$ H/m.)
μ_r	:	Relative permeability, $\mu_r = \mu/\mu_0$.
c	:	Speed of light in vacuo = $(\mu_0\varepsilon_0)^{-\frac{1}{2}}$ m/s.

Constitutive relations

$$\underline{B} = \mu_0(\underline{H} + \underline{M}), \quad \underline{D} = \varepsilon_0\underline{E} + \underline{P}$$

For linear materials:

$$\underline{B} = \mu\underline{H}, \quad \underline{D} = \varepsilon\underline{E}, \quad \underline{J} = \sigma\underline{E}.$$

3.2 Maxwell's equations (integral form).

$$\oint_C \underline{E} \cdot \underline{d\ell} = -\int_S \underline{\dot{B}} \cdot \underline{dS}, \quad \text{Maxwell-Faraday equation.}$$

$$\oint_C \underline{H} \cdot \underline{d\ell} = \int_S (\underline{J} + \underline{\dot{D}}) \cdot \underline{dS}, \quad \text{Maxwell-Ampere equation; also generalised circuital law.}$$

S is any open surface bounded by a closed curve C.

3.3 Gauss's Laws (associated with Maxwell's equations).

$$\oint_S \underline{D} \cdot \underline{dS} = Q, \quad \text{charge enclosed within closed surface } S,$$

$$\oint_S \underline{B} \cdot \underline{dS} = 0 \quad \text{always.}$$

3.4 Maxwell's equations (point form).

Curl of field intensity:

$$\nabla \times \underline{E} = -\underline{\dot{B}},$$
$$\nabla \times \underline{H} = \underline{J} + \underline{\dot{D}}.$$

3.5 Gauss's laws (point form).

Divergence of flux density:

$$\nabla \cdot \underline{D} = \rho,$$
$$\nabla \cdot \underline{B} = 0.$$

3.6 Gradient law.

Gradient of potential:

$$\underline{E} = -\nabla V, \text{ (electrostatics only)},$$
$$\doteq -\left(\underline{i}\frac{\partial V}{\partial x} + \underline{j}\frac{\partial V}{\partial y} + \underline{k}\frac{\partial V}{\partial z}\right) \text{ in Cartesian coordinates.}$$

3.7 Poisson's equation.

$$\nabla^2 V = -\rho/\varepsilon,$$

(becomes Laplace's equation for $\rho = 0$).

3.8 Biot-Savart Law.

$$dH = I\,d\ell\,\sin\theta/4\pi\,r^2 \qquad \text{(scalar notation)}$$

$$\text{or } d\underline{H} = I\,d\underline{\ell} \times \hat{\underline{r}}/4\pi\,r^2$$

(vector notation, where $\hat{\underline{r}}$ is the unit vector in the direction of $\underline{r}$).

3.9 Laplace's Equation.

In rectangular coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

In cylindrical coordinates

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

In spherical coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

3.10 Transmission Lines.

3.10.1 Lossless Transmission Lines

L = loop inductance / unit length.
C = shunt capacitance / unit length.
Wave velocity = $u = \frac{1}{\sqrt{LC}}$.
Wavelength $\lambda = \frac{2\pi u}{\omega}$.
Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}}$.
Phase constant $\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u}$.
Propagation constant = $j\beta$.

For sinusoidal time variation, with forward and reverse waves superposed,

$$V(x,t) = \left(Ae^{-j\beta x} + Be^{j\beta x}\right)e^{j\omega t}$$

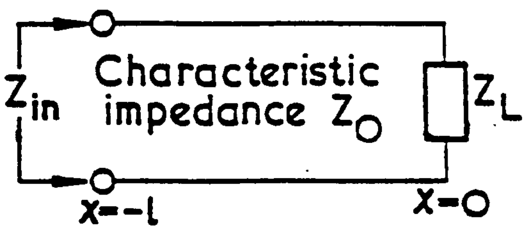
$$I(x,t) = \frac{1}{Z_0}\left(Ae^{-j\beta x} - Be^{j\beta x}\right)e^{j\omega t}$$

At $x = 0$, with load Z_L, voltage reflection coefficient

$$\rho_L = B/A = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L/Z_0 = \frac{1+\rho_L}{1-\rho_L}$$

For any value of x, $\rho(x) = \rho_L e^{j2\beta x}$.



Input impedance $Z_{in} = Z_0 \cdot \dfrac{Z_L + Z_0 j\tan\beta\ell}{Z_0 + Z_L j\tan\beta\ell}$

3.10.2 Lossy Transmission Lines

R = loop resistance / unit length.
G = shunt conductance / unit length.
Characteristic impedance $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$.
Propagation constant $= \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$.

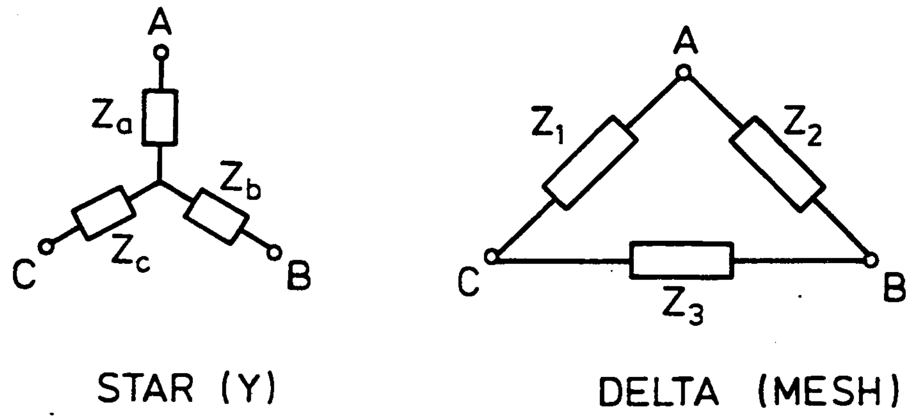
$$V(x,t) = \left(Ae^{-\gamma x} + Be^{\gamma x}\right)e^{j\omega t}$$
$$I(x,t) = \frac{1}{Z_0}\left(Ae^{-\gamma x} - Be^{\gamma x}\right)e^{j\omega t}$$

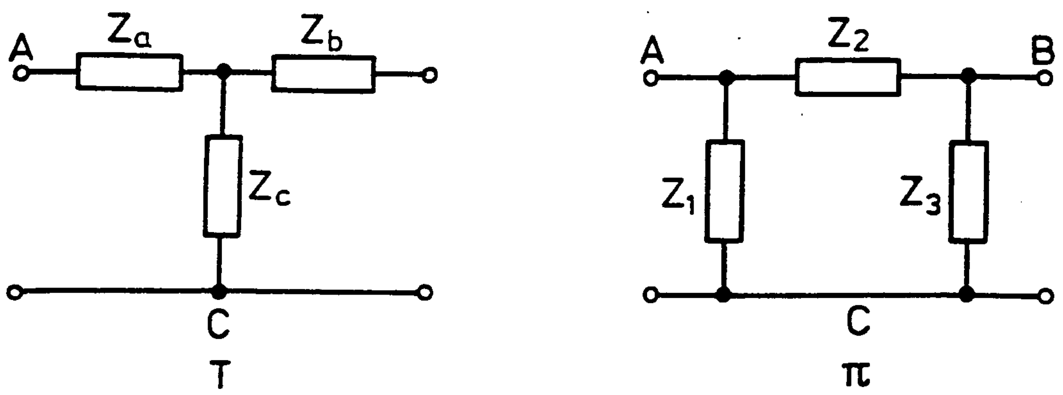
$$Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell}$$

$\rho(x) = \rho_L e^{2\gamma x}$.

4 CIRCUITS INCLUDING LOGIC

4.1 Star-Delta Transformation (Y - mesh or T - π)



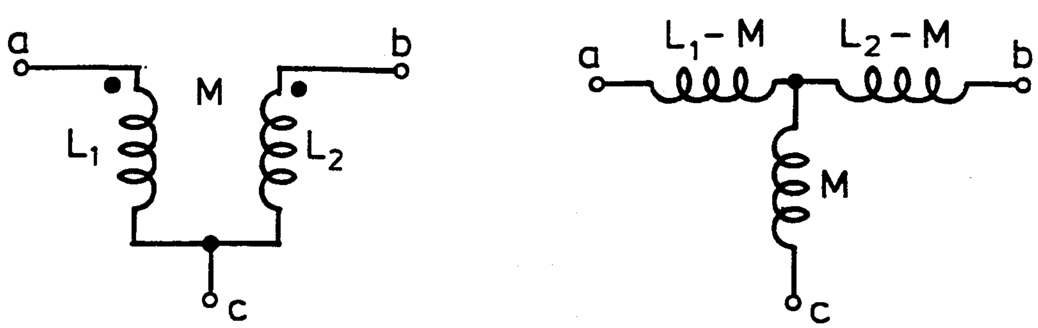


$$Z_a = Z_1 Z_2/(Z_1 + Z_2 + Z_3) \qquad Z_1 = Z_c + Z_a + \frac{Z_c Z_a}{Z_b}$$

$$Z_b = Z_2 Z_3/(Z_1 + Z_2 + Z_3) \qquad Z_2 = Z_a + Z_b + \frac{Z_a Z_b}{Z_c}$$

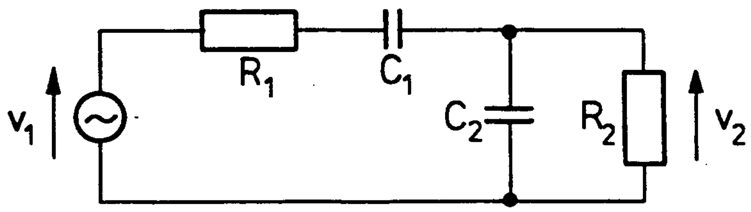
$$Z_c = Z_3 Z_1/(Z_1 + Z_2 + Z_3) \qquad Z_3 = Z_b + Z_c + \frac{Z_b Z_c}{Z_a}$$

4.2 Tee-equivalent of Coupled Coils



4.3 Coupling Circuits

C_1 is usually relatively large and C_2 small.



At midband, when the effects of C_1 and C_2 can be ignored,

$$v_2 = v_1 R_2 / (R_1 + R_2).$$

At high frequencies, v_2 drops to 70% of the midband value when $1/\omega_2 C_2 = R_1 R_2/(R_1 + R_2)$.
At low frequencies, v_2 drops to 70% of the midband value when $1/\omega_1 C_1 = R_1 + R_2$.

ω_1 and ω_2 are known as the lower and upper half power angular frequencies. They are also the -3dB, turnover or 45° phase shift angular frequencies which are other names to describe the same condition.

4.4 Resonant Circuits

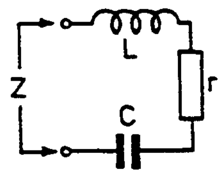
Undamped resonant angular frequency, ω_0, is given by $\omega_0^2 LC = 1$.

Quality factor $Q = \omega U/P$ where U is the total stored energy in the system, and P is the mean power dissipation.

Series resonant circuit:

$$Q_0 = \frac{\omega_0 L}{r} = \frac{1}{r\omega_0 C}, \qquad Z \approx r\left(1 + 2jQ_0\frac{\delta\omega}{\omega_0}\right)$$

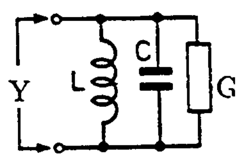
at frequencies close to resonance



Parallel resonant circuit:

$$Q_0 = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G} \qquad Y \approx G\left(1 + 2jQ_0\frac{\delta\omega}{\omega_0}\right)$$

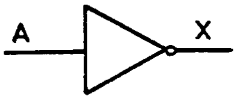
at frequencies close to resonance



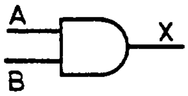
For series resonant circuit, $Z = r(1 \pm j)$ when $\delta\omega/\omega_0 = \pm 1/(2Q_0)$.

Half power bandwidth $= (1/Q)\times$ resonant frequency.

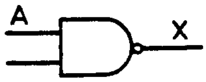
4.5 Logic



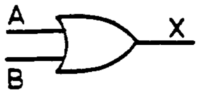
COMPLEMENT : $X = \overline{A}$



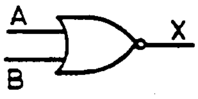
AND: $X = A.B$



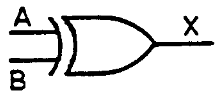
NAND : $X = \overline{A.B}$



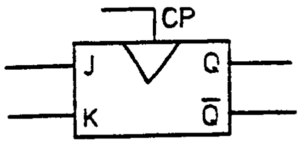
OR : $X = A+B$



NOR: $X = \overline{A+B}$



EXCLUSIVE OR : $X = A\overline{B}+\overline{A}B$



J K Flip - Flop

Excitation table			
Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	1	X	0
1	0	X	1

Truth table		
J_n	K_n	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\overline{Q}_n$

In the excitation table X represents a 'don't care' state.

4.6 Boolean Algebra

Commutation

$$A + B = B + A$$
$$A \cdot B = B \cdot A$$

Association

$$A + (B + C) = (A + B) + C$$
$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Absorption

$$A + (A \cdot B) = A$$
$$A \cdot (A + B) = A$$

Distribution

$$A \cdot (B + C) = A \cdot B + A \cdot C$$
$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

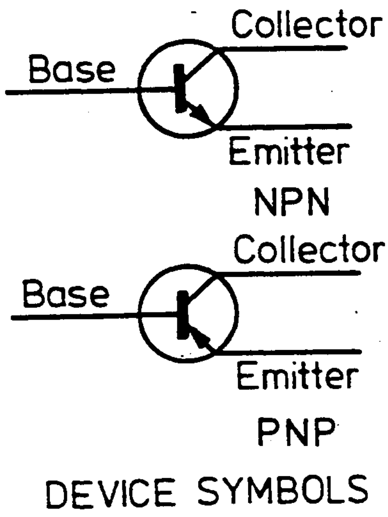
De Morgan's Theorems

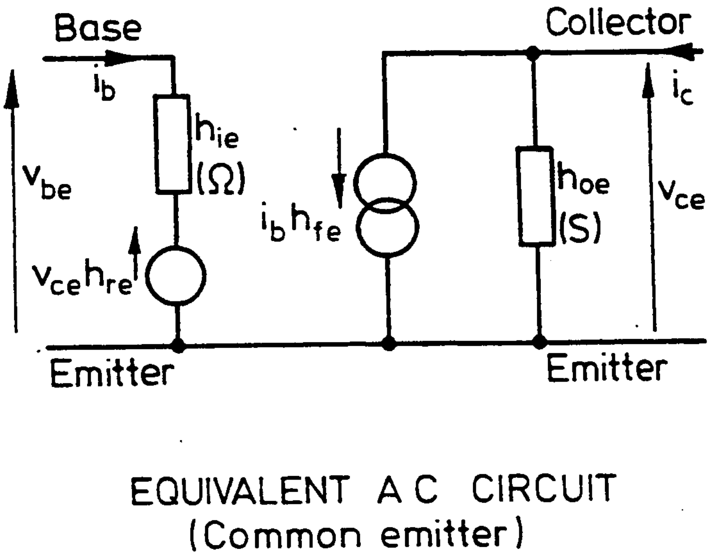
$$\overline{A + B} = \overline{A} \cdot \overline{B} \qquad \overline{A \cdot B} = \overline{A} + \overline{B}$$

where the bar indicates the inversion operation, $\overline{A}$ meaning 'not A'.

5 SMALL SIGNAL EQUIVALENT CIRCUITS OF TRANSISTORS OPERATING AT LOW FREQUENCIES, INCLUDING THE OPERATIONAL AMPLIFIER

5.1 Bipolar Transistors





The currents and voltages shown on the diagram are small a.c. signals superimposed upon a specified d.c. operating condition. The hybrid parameters (h) depend on the d.c. operating condition and relate the signals by these equations:

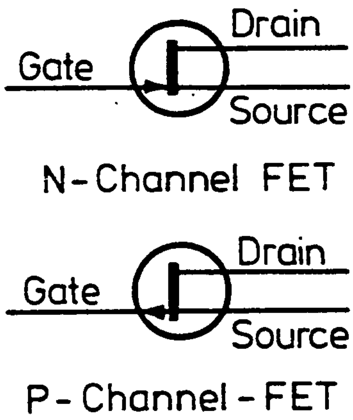
$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$
$$i_c = h_{fe} i_b + h_{oe} v_{ce}$$

Typical values for a BC108B at 1 kHz, junction temperature of 25 °C and at the d.c. operating conditions of $I_C = 2\ mA$, $V_{CE} = 5$ V are:

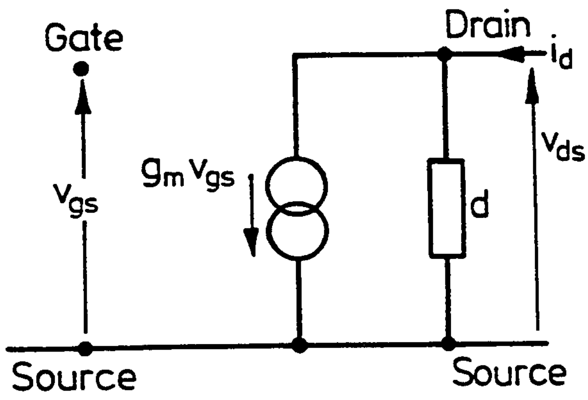
h_{ie}; 3.2 to 8.5 $k\Omega$: h_{re}; 2×10^{-4}
h_{fe}; 240 to 500 : h_{oe}; 30 to 100 μS.

h_{re} and h_{oe} often may be neglected.

5.2 Junction Field-Effect Transistors.



DEVICE SYMBOLS



EQUIVALENT A.C. CIRCUIT

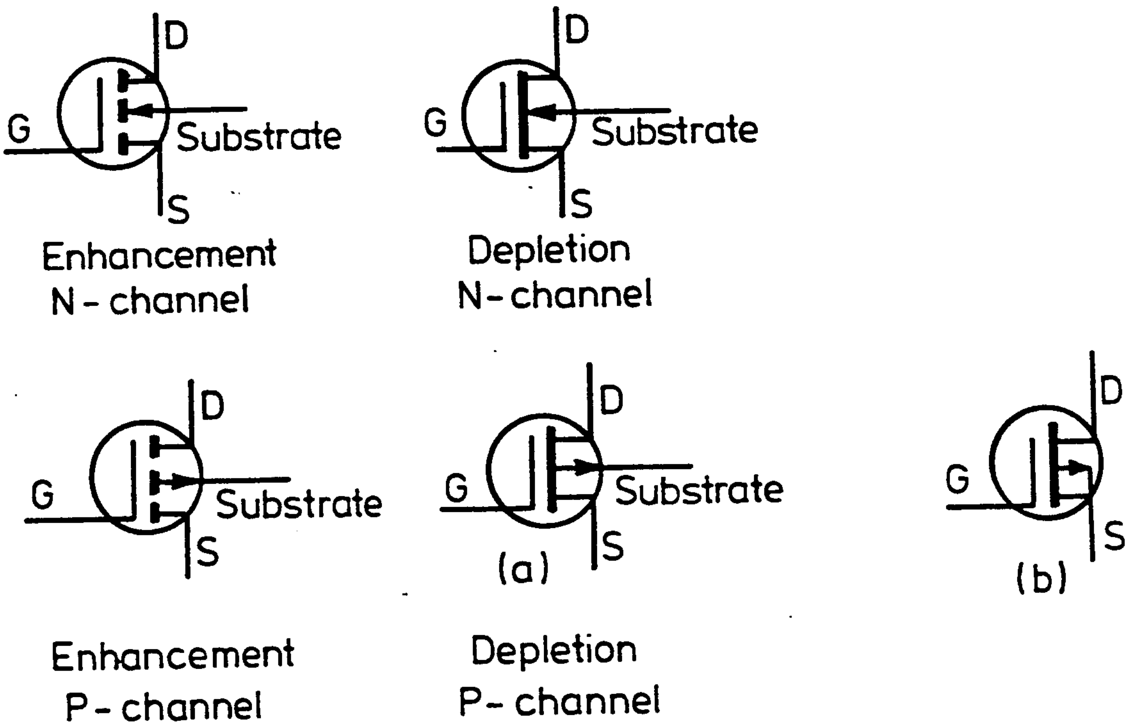
The currents and voltages shown on the diagram are small a.c. signals superimposed upon a specified d.c. operating condition. The device parameters depend on the d.c. operating condition and relate the signals by this equation:

$$i_d = g_m v_{gs} + v_{ds}/r_d.$$

Typical values for the low-power silicon transistor $2N3819$ are:

$$g_m;\ 2 - 6.5\ mS:\quad r_d; > 20\ k\Omega.$$

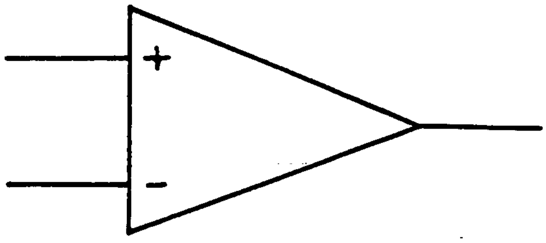
5.3 Insulated Gate Field-Effect Transistors (including MOSFETs)



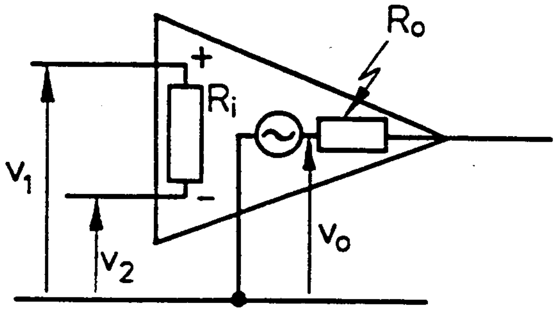
In many cases the substrate is internally connected to the source electrode and the symbol for the p-channel depletion FET is as shown in Fig. (b).

The circuit model is as for the junction FET.

5.4 Operational Amplifier



Symbol



Circuit model

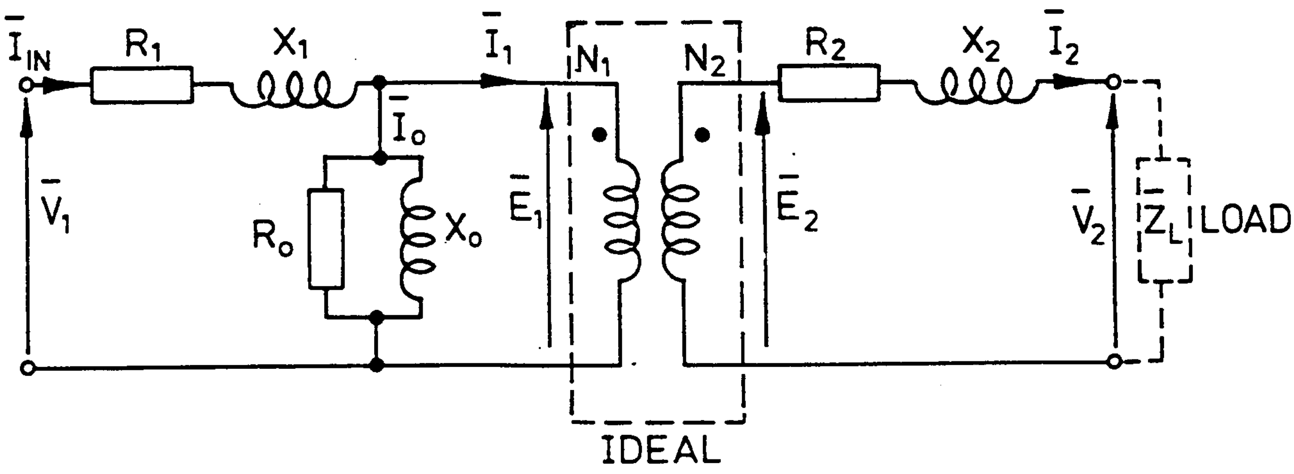
$$v_o = A(v_1 - v_2) + A_{cm}\frac{v_1 + v_2}{2}$$

A is the differential gain and A_{cm} is the common mode gain. Frequently A_{cm} may be neglected for the purpose of analysis.

6 ELECTRICAL POWER AND MACHINES

6.1 Transformer

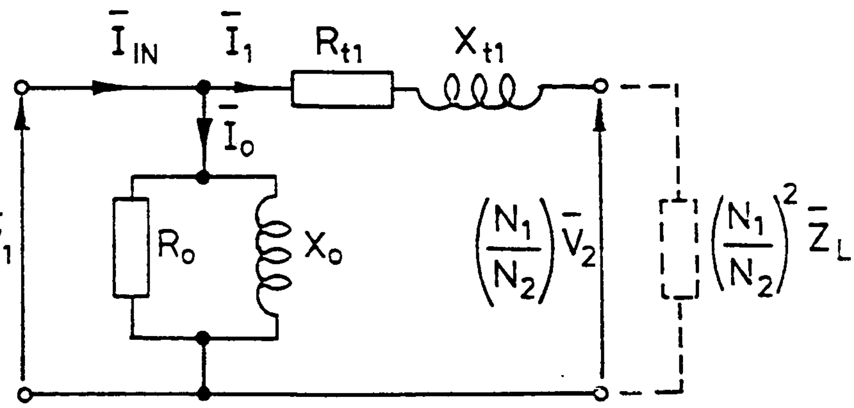
6.1.1 Complete equivalent circuit



$$\bar{I}_1 N_1 = \bar{I}_2 N_2$$

$$\frac{\bar{E}_1}{N_1} = \frac{\bar{E}_2}{N_2}$$

6.1.2 Simplified Equivalent Circuit

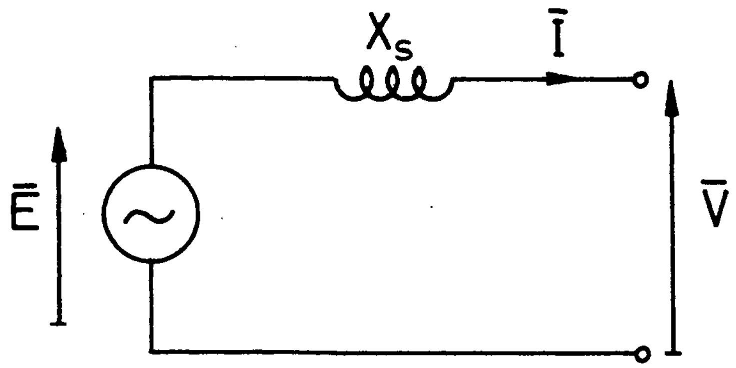


$$R_{t1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2$$

$$X_{t1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2$$

6.2 Three-phase synchronous machine

6.2.1 Equivalent circuit for cylindrical rotor machine (motor)



$$V = \text{Terminal phase voltage}$$
$$I = \text{Input phase current}$$
$$X_s = \text{Synchronous reactance (per phase)}$$
$$E = \text{Generated phase emf.}$$

6.2.2 Basic relationships

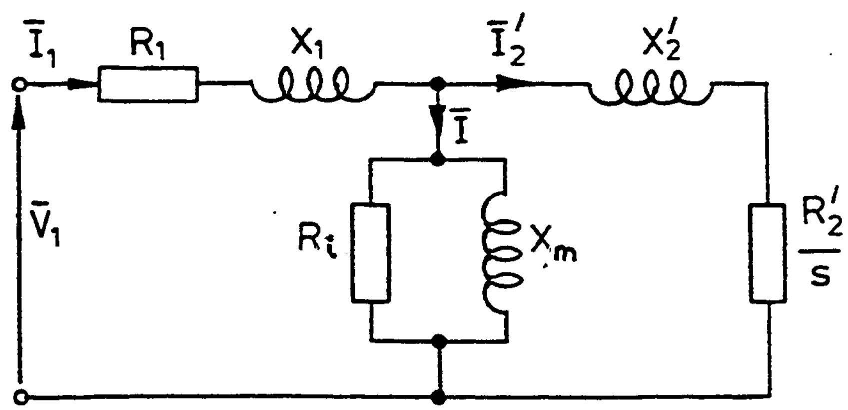
$$\omega_s = \text{Synchronous speed (rad/s)}$$
$$\omega = \text{Supply frequency (rad/s)}$$
$$p = \text{Number of pole-pairs}$$
$$\delta = \text{Electrical load angle.}$$

$$\omega_s = \frac{\omega}{p}$$

$$\text{Electromagnetic torque, } T = \frac{3VE}{\omega_s X_s} \sin\delta$$

6.3 Three-phase induction motor

6.3.1 Equivalent circuit



6.3.2 Basic equations

$$\text{Slip, } s = \frac{\omega_s - \omega_r}{\omega_s}$$

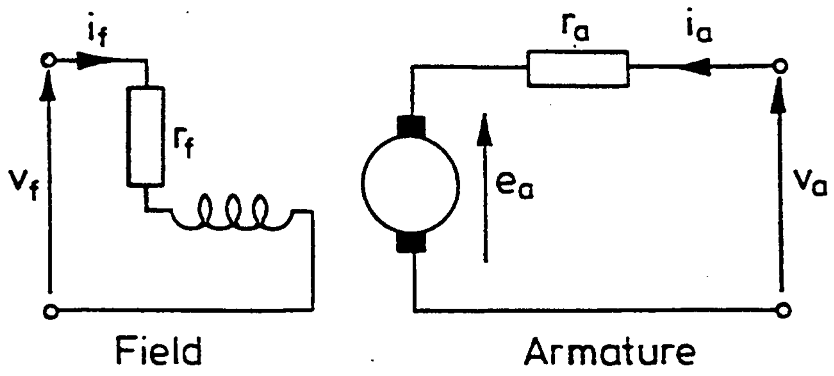
$$\text{Total torque, } T = \frac{3\overline{I}'^2_2}{\omega_s}\frac{R'_2}{s}$$

ω_s = synchronous speed as defined in 6.2.2.

ω_r = Rotor speed (rad/s)

6.4 Separately-excited d.c. motor

6.4.1 Equivalent circuit for separately-excited motor



6.4.2 Basic relationships

$$e_a = K \phi \omega$$
$$T = K \phi i_a$$

K = emf constant

ϕ = flux per pole $\quad \phi = \phi(i_f)$

ω = rotor speed

i_a = armature current

T = torque

6.5 Per-Unit Calculations

6.5.1 VA_b = Base VA (three-phase)

V_b = Base line voltage

$$Z_b = \frac{V_b^2}{VA_b}$$

$$I_b = \frac{VA_b}{\sqrt{3}V_b}$$

6.5.2 Change of base VA

$$Z_{pu(2)} = Z_{pu(1)} \frac{VA_{b(2)}}{VA_{b(1)}}$$

7 FOURIER SERIES ANALYSIS OF PERIODIC WAVEFORMS

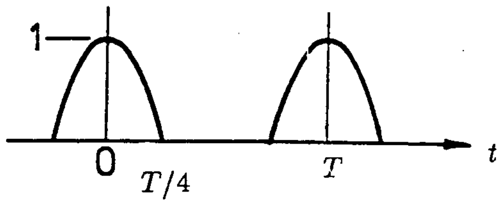
If $g(t)$ is periodic over $-T/2$ to $T/2$ then:

$$\begin{aligned} g(t) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right] \\ \text{where} \quad a_n &= \frac{2}{T}\int_{-T/2}^{T/2} g(t)\cos(n\omega_0 t)\,dt \quad \text{and} \quad b_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\sin(n\omega_0 t)\,dt \end{aligned}$$

or:

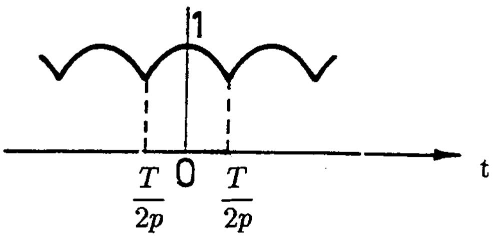
$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad \text{where} \quad c_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)e^{-jn\omega_0 t}\,dt = \frac{a_n - jb_n}{2}$$

Where $\omega_0 = 2\pi/T = 2\pi f_0$; $f_0 = 1/T$ is the fundamental frequency.



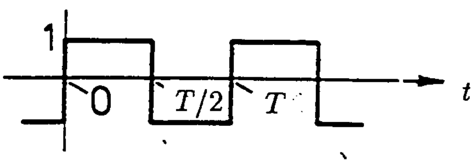
Half-wave rectified cosine wave:

$$g(t) = \frac{1}{\pi} + \frac{1}{2}\cos(\omega_0 t) + \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos(2n\omega_0 t)}{4n^2-1}$$



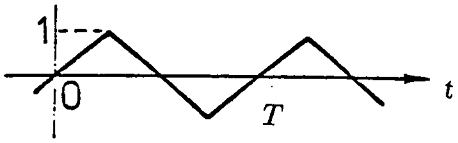
p-phase rectified cosine wave ($p \geq 2$):

$$g(t) = \frac{p}{\pi}\sin\frac{\pi}{p}\left[1 + 2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos(pn\omega_0 t)}{p^2n^2-1}\right]$$



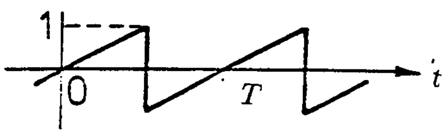
Square wave:

$$g(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)\omega_0 t}{2n-1}$$



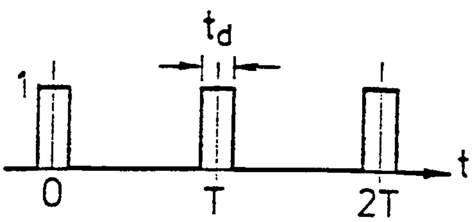
Triangular wave:

$$g(t) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(2n-1)\omega_0 t}{(2n-1)^2}$$



Sawtooth wave:

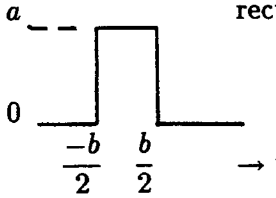
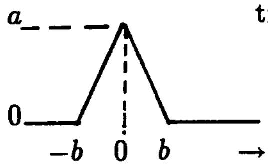
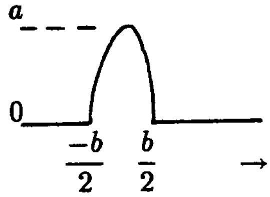
$$g(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(n\omega_0 t)}{n}$$



Pulse wave:

$$g(t) = \frac{t_d}{T}\left[1 + 2\sum_{n=1}^{\infty}\frac{\sin(n\pi t_d/T)}{(n\pi t_d/T)}\cos(n\omega_0 t)\right]$$

8 TABLE OF FOURIER TRANSFORM RELATIONS

Waveform: $g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}\,d\omega$		Spectrum $(\omega = 2\pi f)$: $G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt$	
1	DC level	$2\pi\,\delta(\omega) = \delta(f)$	
$u(t)$	unit step	$\pi\,\delta(\omega) + \frac{1}{j\omega}$	
$e^{j\omega_0 t}$		$2\pi\,\delta(\omega - \omega_0)$	
$\cos(\omega_0 t)$		$\pi\,[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
$\sin(\omega_0 t)$		$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	impulse train	$\frac{2\pi}{T}\sum_{m=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi m}{T}\right)$	
	rectangular pulse	$ab\,\text{sinc}\left(\frac{\omega b}{2}\right)$ Note: $\text{sinc}(x) = \frac{\sin(x)}{x}$	
	triangular pulse	$ab\,\text{sinc}^2\left(\frac{\omega b}{2}\right)$	
	half-sine pulse	$\frac{ab}{2}\left[\text{sinc}\left(\frac{\omega b - \pi}{2}\right) + \text{sinc}\left(\frac{\omega b + \pi}{2}\right)\right]$	
$g(t - t_0)$	time shift	$e^{-j\omega t_0}\,G(\omega)$	
$e^{j\omega_0 t}\,g(t)$		$G(\omega - \omega_0)$	frequency shift
$\frac{d^n g(t)}{dt^n}$	differentiation	$(j\omega)^n G(\omega)$	
$g_1(t) * g_2(t) = \int_{-\infty}^{\infty} g_1(t - \tau)g_2(\tau)\,d\tau$	convolution	$G_1(\omega)\,G_2(\omega)$	
$g_1(t)\,g_2(t)$	multiplication	$\frac{1}{2\pi}G_1(\omega) * G_2(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(\omega - \Omega)G_2(\Omega)\,d\Omega$	

Duality: If $g(t)$ transforms to $p(\omega)$, then $p(t)$ transforms to $2\pi g(-\omega)$.

Symmetry: If $g(t)$ is real, then $G(-\omega) = G^*(\omega)$ (* means complex conjugate).
If $g(t)$ is real and even, then $G(\omega)$ is real and even.
If $g(t)$ is real and odd, then $G(\omega)$ is imaginary and odd.

Parseval's theorem of energy conservation: $\int_{-\infty}^{\infty} |g(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2\,d\omega$

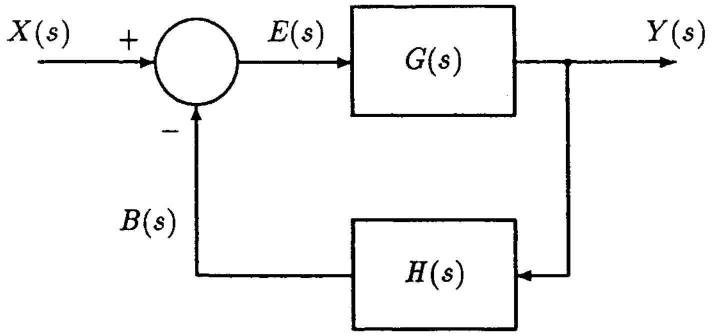
9 TABLE OF LAPLACE TRANSFORM RELATIONS

Waveform: $g(t)$ (defined for $t \geq 0$)		Laplace Transform: $G(s) = \mathcal{L}\{g(t)\} = \int_{0_-}^{\infty} g(t)e^{-st}\,dt$	
$\delta(t)$	impulse	1	
$u(t)$	unit step	$\frac{1}{s}$	
t^n		$\frac{n!}{s^{n+1}}$	
e^{-at}		$\frac{1}{s+a}$	
$\sin(\omega_0 t)$		$\frac{\omega_0}{s^2+\omega_0^2}$	
$\cos(\omega_0 t)$		$\frac{s}{s^2+\omega_0^2}$	
$\sinh(\omega_0 t)$		$\frac{\omega_0}{s^2-\omega_0^2}$	
$\cosh(\omega_0 t)$		$\frac{s}{s^2-\omega_0^2}$	
$e^{-at}[A\cos(\omega_0 t) + B\sin(\omega_0 t)]$		$\frac{A(s+a)+B\omega_0}{(s+a)^2+\omega_0^2}$	
$e^{-at}g(t)$		$G(s+a)$	shift in s
$g(t-\tau)u(t-\tau)$ where $\tau \geq 0$		$e^{-s\tau}G(s)$	shift in t
$t\,g(t)$		$-\frac{d}{ds}G(s)$	
$\frac{dg}{dt}$	differentiation	$sG(s) - g(0)$	
$\frac{d^n g}{dt^n}$		$s^n G(s) - s^{n-1}g(0) - s^{n-2}\left(\frac{dg}{dt}\right)_0 - \cdots - \left(\frac{d^{n-1}g}{dt^{n-1}}\right)_0$	
$\int_0^t g(\tau)\,d\tau$	integration	$\frac{G(s)}{s}$	
$g_1(t) * g_2(t) = \int_0^t g_1(t-\tau)g_2(\tau)\,d\tau$	convolution	$G_1(s)G_2(s)$	

10 TABLE OF Z-TRANSFORM RELATIONS

Sequence: $g_k, \quad k = 0, 1, 2, \cdots$	z Transform: $G(z) = \sum_{k=0}^{\infty} g_k z^{-k}$
1 (unit step)	$\frac{1}{1 - z^{-1}}$
kT	$\frac{Tz^{-1}}{(1 - z^{-1})^2}$
$\frac{(k+m-1)!}{k!\,(m-1)!}$	$\frac{1}{(1 - z^{-1})^m}$
e^{-akT}	$\frac{1}{1 - e^{-aT} z^{-1}}$
$\sin(\omega_0 kT)$	$\frac{\sin(\omega_0 T)\, z^{-1}}{1 - 2\cos(\omega_0 T)\, z^{-1} + z^{-2}}$
$\cos(\omega_0 kT)$	$\frac{1 - \cos(\omega_0 T)\, z^{-1}}{1 - 2\cos(\omega_0 T)\, z^{-1} + z^{-2}}$
$\frac{r^{k-1}}{\sin \omega_0 T} [r \sin(\omega_0 (k+1) T) - a \sin(\omega_0 kT)]$	$\frac{1 - az^{-1}}{1 - 2r\cos(\omega_0 T)\, z^{-1} + r^2 z^{-2}}$
$r^k [A\cos(\omega_0 kT) + B\sin(\omega_0 kT)]$	$\frac{A + rz^{-1}(B\sin(\omega_0 T) - A\cos(\omega_0 T))}{1 - 2r\cos(\omega_0 T)\, z^{-1} + r^2 z^{-2}}$
$r^k g_k$	$G(r^{-1} z)$
g_{k+1}	$zG(z) - zg_0$
g_{k-1}	$z^{-1}G(z) + g_{-1}$
g_{k+m}	$z^m G(z) - z^m g_0 - \cdots - zg_{m-1}$
g_{k-m}	$z^{-m}G(z) + z^{-(m-1)} g_{-1} + \cdots + g_{-m}$
$g_0 = \lim_{z \to \infty} G(z)$	(initial value theorem)
$\lim_{k \to \infty} g_k = \lim_{z \to 1} (z-1) G(z)$	(final value theorem when poles of $(z-1)G(z)$ are inside unit circle)

11 CONTROL



Loop Transfer Function: $\dfrac{B(s)}{E(s)} = G(s)H(s)$.

Closed-loop Transfer Function: $\dfrac{Y(s)}{X(s)} = \dfrac{G(s)}{1 + G(s)H(s)} = \dfrac{G(s)}{1 + K\,g(s)}$.

where it is convenient to define $G(s)H(s) = K\,g(s)$, an explicit function of loop gain K.

Stability of the Closed-loop System:

The closed-loop system is stable if the roots of the characteristic equation, $1 + K\,g(s) = 0$, have negative real parts.

Routh-Hurwitz Stability Criteria:

If $1 + K\,g(s)$ is n^{th} order and is expressed as the ratio of two polynomials, $A(s)/B(s)$, where $A(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0$, then the closed-loop system is stable:

for $n = 2$, if all $a_i > 0$;
for $n = 3$, if all $a_i > 0$ and $a_1 a_2 > a_0 a_3$;
for $n = 4$, if all $a_i > 0$ and $a_1 a_2 a_3 > a_0 a_3^2 + a_4 a_1^2$.

(Further relationships exist for $n > 4$.)

Nyquist Stability Criterion:

For a stable closed-loop system, the full Nyquist plot of $g(s)$, for $s = j\omega$ and $-\infty < \omega < \infty$, should encircle the $(-\frac{1}{K}, j0)$ point as many times as there are poles of $g(s)$ (i.e. open-loop poles) in the right half of the s-plane. The encirclements, for the path traced by increasing ω, must be in a counterclockwise direction.

Root Locus:

The roots of $1 + K\,g(s) = 0$, the closed loop poles, trace loci as K varies from 0 to ∞, starting at the open-loop poles and ending at the open-loop zeros or at infinite distances. All sections of the real axis with an odd number of singularities to their right are sections of the root locus. At the breakaway points (coincident roots): $dK/ds = 0$

Angle condition: $\angle g(s) = (2m + 1)\pi$ (where m is an integer).

Magnitude condition: $|g(s)| = 1/K$

Asymptotes: If $g(s)$ has P poles and Z zeroes, the asymptotes of the loci as $K \to \infty$ are straight lines at angles $(2m + 1)\pi/(P - Z)$ to the real axis. Their point of intersection σ with the real axis is given by: $\sigma = \left(\sum \text{poles of } g(s) - \sum \text{zeroes of } g(s)\right)/(P - Z)$

12 COMMUNICATIONS

Shannon's Law for Channel Capacity:

The maximum bit rate C at which information may be transmitted over a channel of bandwidth B Hz and with a mean signal power S and noise power N is given by:

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bit/s.}$$

Amplitude Modulation:

The modulated signal $s(t)$ is related to the modulating signal $x(t)$ by:

$$s(t) = [a_0 + x(t)]\cos(\omega_C t + \phi)$$

$$\text{Modulation index, } m_A = \frac{\text{Peak amplitude of } x(t)}{a_0}$$

Frequency Modulation:

The modulated signal $s(t)$ is related to the modulating signal $x(t)$ by:

$$s(t) = a_0\cos[\omega_C t + \phi(t)] \quad \text{where } \frac{d\phi}{dt} = K_F x(t) \text{ rad/s.}$$

$$\text{Frequency deviation, } f_D = \frac{K_F}{2\pi}[\text{Peak amplitude of } x(t)] \text{ Hz}$$

If $x(t) = a_x\cos(\omega_M t)$ where the modulating frequency $f_M = \frac{\omega_M}{2\pi}$, then:

$$\text{Modulation index, } m_F = \frac{f_D}{f_M} = \frac{K_F a_x}{\omega_M}$$

Carson's Rule for FM signals:

$$\text{Modulated signal bandwidth} \approx 2(m_F+1)f_M = 2(f_D+f_M)$$