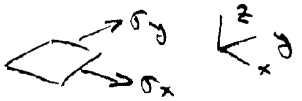
9 $\therefore\ \sigma_x = \sigma_y = 2\ \text{MPa}, \quad \sigma_z = 0$



$$\therefore\ \epsilon_z = \tfrac{1}{E}\left[\sigma_z - \nu(\sigma_x+\sigma_y)\right] = -\tfrac{2\nu}{E}\sigma_x$$

Also $G = E/2(1+\nu)$

or $\nu = E/2G - 1 = 3.5/2\times1.4 - 1 = 0.25$

$$\therefore\ \epsilon_z = -\frac{2\times0.25}{3.5\times10^3}\times 2 = -2.86\times10^{-4}$$

$= (\delta/L)_z$ where $L_z = 0.7$ mm

$$\therefore\ \delta_z = (\epsilon \cdot L)_z = -2.86\times10^{-4}\times0.7 = \underline{2\times10^{-4}}\ \text{mm}$$

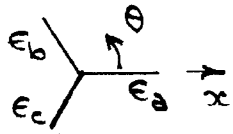
10 Approach: from tension test with known stress, the measured strain enables the elastic properties (E, ν) to be determined. These then enable measured strains on the component to be transformed into stresses.

Specimen – from 2-D stress-strain relations

longitudinal $\epsilon = \sigma/E \ \therefore E = 125/604\times10^{-6} = 207$ GPa

transverse $\epsilon_y = -\nu\sigma_x/E \ \therefore \nu = 177/604 = 0.293$

Strain gauge rosette The equiangular rosette gives three normal strain components, say $\epsilon_a, \epsilon_b, \epsilon_c$ at $\theta_a = 0°$, $\theta_b = 120°$, $\theta_c = -120°$ as shown.



We wish to deduce the basic components from given $\epsilon_{a,b,c}$. Thus, from 4b:

$$\epsilon_a = \bar\epsilon + \tilde\epsilon\cos 2\theta_p$$
$$\epsilon_b = \bar\epsilon + \tilde\epsilon\cos 2(\theta_p - 120°) = \bar\epsilon + \tfrac12\tilde\epsilon(-\cos 2\theta_p - \sqrt3\sin 2\theta_p)$$
$$\epsilon_c = \bar\epsilon + \tilde\epsilon\cos 2(\theta_p + 120°) = \bar\epsilon + \tfrac12\tilde\epsilon(-\cos 2\theta_p + \sqrt3\sin 2\theta_p)$$

Solving for the basic triad:

$$\bar\epsilon = (\epsilon_a + \epsilon_b + \epsilon_c)/3$$
$$\tilde\epsilon\sin 2\theta_p = (\epsilon_c - \epsilon_b)/\sqrt3$$
$$\tilde\epsilon\cos 2\theta_p = (2\epsilon_a - \epsilon_b - \epsilon_c)/3$$

With $\epsilon_a = -410$, $\epsilon_b = 655$, $\epsilon_c = -40\times10^{-6}$, this gives $\bar\epsilon = 68.3$, $\tilde\epsilon = 624.3\times10^{-6}$, $\theta_p = -70°$.

Strain-stress relations – from (5)

$$\left.\begin{aligned}\bar\sigma &= E\bar\epsilon/(1-\nu) = 20\ \text{MPa}\\ \tilde\sigma &= E\tilde\epsilon/(1+\nu) = 100\ \text{MPa}\end{aligned}\right\}$$ using derived E, ν values above.

So $\check\sigma, \hat\sigma = 20 \pm 100 = -80, \underline{120}$ MPa.