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Approach: similar to previous problem – find section with most critical M_E. A vector analysis of the loads is more appropriate than 2-D resolution. Thus

$M = r \times F$ i.e. $[M_x\ M_y\ M_z] = [x\ y\ z] \times [F_x\ F_y\ F_z]$.

Stress concentration at the corners b, c, e, f will be ignored, as will be the finite radii of curvature.

Using the co-ordinate system sketched,

$F_a = [-200\ \ 250\ \ 0]$ $F_d = [0\ \ -500\ \ 0]$ N.

$M_b = [0.08\ \ 0\ \ 0] \times F_a = [0\ \ 0\ \ 20]$ Nm

$M_c = [0.08\ \ 0\ \ 0.12] \times F_a = [-30\ \ 24\ \ 20]$ "

$M_d = [0.16\ \ 0\ \ 0.12] \times F_a = [-30\ \ 24\ \ 40]$ "

$M_e = [0.24\ \ 0\ \ 0.12] \times F_a + [0.08\ \ 0\ \ 0] \times F_d = [-30\ \ 24\ \ 20]$ Nm

$M_f = [0.24\ \ 0\ \ 0] \times F_a + [0.08\ \ 0\ \ -0.12] \times F_d = [-60\ \ 0\ \ 20]$ "

$M_g = [0.32\ \ 0\ \ 0] \times F_a + [0.16\ \ 0\ \ -0.12] \times F_d = [-60\ \ 0\ \ 0]$ "

Now a twist about the x-axis (i.e. M_x) represents a torque for shaft portion f-g for example, i.e.

for a-b, c-d-e, f-g $\rightarrow$ $M_x \equiv T$; $\sqrt{M_y^2 + M_z^2} \equiv M$

for b-c, e-f $\rightarrow$ $M_z \equiv T$; $\sqrt{M_x^2 + M_y^2} \equiv M$

The other two components of each M-vector represent the bending moment, calling these components M_1 & M_2

shaft section	ab		bc		cde			ef		fg	
location	a	b	b	c	c	d	e	e	f	f	g
T (Nm)	0	0	20	20	30	30	30	20	20	60	60
M_1 "	0	20	0	24	24	24	24	24	60	20	0
M_2 "	0	0	0	30	20	40	20	30	0	0	0
$M_E = \sqrt{[M_1^2 + M_2^2 + \lambda T^2]}$; $\lambda = 1$	0	20	20	43.3	43.3	55.5	43.3	43.3	63.3	63.3	60
$\lambda = 3/4$	0	20	17.3	42.1	40.6	53.4	40.6	42.1	62.5	55.7	52.0

Note, at 'e' for example, how turning the corner from section cde to ef transposes the 20, 30 moment, torque components. This has no effect on M_E if the max. shear stress theory ($\lambda = 1$) is used. But there is some effect with the distortion energy theory – which it will be noted always predicts a lower M_E (it is less conservative than the max. shear stress theory.)

Location f in ef is the most critical cross-section.

From (ii) $n = ZS/M_E$

$= \frac{\pi}{32} \times 12^3 \times 450 \,/\, 63.3 \times 10^3 = 1.21$ (max shear)

$\quad mm^3 \quad \frac{N}{mm^2} \quad \frac{1}{Nm} \quad \frac{m}{mm}$ or 1.22 (dist^n energy)