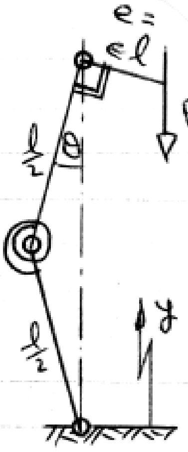

Angle at spring $= \pi - 2\theta$

$\therefore$ spring deformation, $\Delta = (\pi - 2\theta) - (\pi - 2\theta_0) = 2(\theta_0 - \theta)$

Load elevation, $y_P = l\cos\theta - e\sin\theta = l(\cos\theta - \epsilon\sin\theta)$

Energy $= \frac{1}{2}k\,4(\theta_0 - \theta)^2 + Pl[(\cos\theta - \epsilon\sin\theta) - (\cos\theta_0 - \epsilon\sin\theta_0)]$

Set U = energy$/4k$ & $p = Pl/4k$.

$= \frac{1}{2}(\theta - \theta_0)^2 + p[(\cos\theta - \cos\theta_0) - \epsilon(\sin\theta - \sin\theta_0)]$

(1) $U' = \theta - \theta_0 - p(\sin\theta + \epsilon\cos\theta)$

(2) $U'' = 1 - p(\cos\theta - \epsilon\sin\theta)$

PERFECT $\theta_0 = \epsilon = 0$ — critical buckling occurs

$U' = \theta - p\sin\theta = 0$ for equilibrium, i.e.

(3) either $\theta = 0$ — in which case $U'' = 1 - p$.

neutral, $U'' = 0$, $p = 1$ i.e. critical $p_c = 1$, $P_c = 4k/l$.

stable, $U'' > 0$, $p < 1$ i.e. $p < p_c$

unstable, $U'' < 0$, $p > 1$ i.e. $p > p_c$

(4) or $p = \theta/\sin\theta > 1$ (mathematically, for all θ)

For this post-buckling path (i.e. $p > p_c$),

$U'' = 1 - \theta\cot\theta > 0$ for all θ, i.e. inherently stable.

IMPERFECT $\theta \geq \theta_0 > 0$, $\epsilon > 0$

For equilibrium, $U' = 0$, so, from (1)

(5) $p = (\theta - \theta_0)/(\sin\theta + \epsilon\cos\theta)$. — in which case

$U'' = 1 - (\theta - \theta_0)(\cos\theta - \epsilon\sin\theta)/(\sin\theta + \epsilon\cos\theta)$

Postulate that structure is always stable, i.e. $U'' > 0$

so $\tan(\theta + \tan^{-1}\epsilon) > \theta - \theta_0$

or $\theta < \tan(\theta + \tan^{-1}\epsilon) + \theta_0$

Since θ is always less than $\tan\theta$, the above inequality must be satisfied for $\epsilon, \theta_0 > 0$. So postulate is proven correct; always stable.

Though note that if θ_0 and ϵ are of opposite signs, this generalisation does not apply.

Stability is demonstrated by the shape of the paths, (5), shown overleaf. It is apparent from the general shapes of the two sets of curves, for (i) and for (ii), that the general behaviour is similar. This leads to the conclusion that, generally, imperfections cause similar deviations from ideal behaviour, so that it is difficult in practice to deduce the type of imperfection (e.g. lack of straightness, eccentricity of load, etc) from measurements of, say, the equilibrium paths. Hence the concept of "equivalent imperfection".

Note how in structure (i), the path tends to the ideal at large $|\theta|$ as the moment arm aligns with the structural axis.

cont'd . . . .