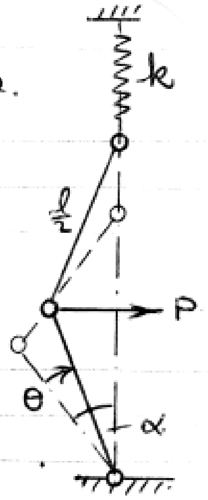

Spring undeformed when inclination α, $(P = 0)$
Displacement θ as shown from undeformed state.
Set $\phi = \alpha - \theta$; $\phi' = -1$; $' \equiv d/d\theta$.
Load displacement $= \frac{\ell}{2}(\sin\alpha - \sin\phi)$.
Spring deflexion $= \ell(\cos\phi - \cos\alpha)$
$\therefore$ Energy $= \frac{1}{2}k\ell^2(\cos\phi - \cos\alpha)^2 + P\frac{1}{2}\ell(\sin\phi - \sin\alpha)$
Set V $=$ energy$/k\ell^2$; & $\underline{p = P/2k\ell}$

$$= \tfrac{1}{2}(\cos\phi - \cos\alpha)^2 + p(\sin\phi - \sin\alpha)$$

$$\therefore \quad V' = (\cos\phi - \cos\alpha)\sin\phi - p\cos\phi$$

$$V'' = -\cos 2\phi + \cos\alpha\cos\phi - p\sin\phi$$

For equilibrium, $V' = 0$, so

① $$p = (1 - \cos\alpha/\cos\phi)\sin\phi \quad ; \quad \phi = \alpha - \theta$$

$$V'' = (\cos\alpha - \cos^3\phi)/\cos\phi$$

- from which it may be deduced that if $\cos^3\phi > \cos\alpha$ i.e. if $9.4° < \theta < 35.6°$ (with $\alpha = 22.5°$), $V'' < 0$ and the structure is unstable. If $\theta < 9.4°$ or $\theta > 35.6°$ the structure is stable.

Neutral equilibrium occurs when $\theta = 9.4°$ or $35.6°$ i.e. when $\cos\phi = (\cos\alpha)^{1/3}$, or, when

$$\cos\theta = \cos^{4/3}\alpha \pm \sin\alpha(1 - \cos^{2/3}\alpha)^{1/2}$$

corresponding to local minima/maxima given by ① as:

$$\hat{\check{p}} = \pm(1 - \cos^{2/3}\alpha)^{3/2} = \pm 0.0117$$

The above characteristics are confirmed by the plot of ① vs. θ, shown overleaf for $\alpha = 22.5°$.
Consider what happens to the structure as both the displacement θ and the load p (P) increase from zero under two different physical constraints:–

Displacement controlled.

Here the loading device is arranged to increase θ monotonically and to apply the necessary force, as dictated by the structure's characteristic, automatically. Hence the full-line characteristic is traced out.

Load controlled.

In this arrangement, which is the simplest and most common, the loading device increases the force monotonically, the structure being allowed to assume whatever geometry (displacement) it wishes. As this load is increased from zero to the point A, an infinitesimal increase causes the dashed trajectory A-B to be undergone, with consequent instantaneous increase in displacement. This highly non-linear instability (temporary) is known as SNAP. Further increase of load beyond B is uneventful.

cont'd....