12 Extending eq(4) of notes to cover expansion $\alpha.\delta t$ – as foreshadowed in (X), $\alpha.\delta t$ must be added to strain.
$\therefore \delta_i/D_c = [2p_i - \{(1-\nu)\gamma_0+1+\nu\}p_c]/E(\gamma_0-1) + \alpha_i\,\delta T_i$ – inner
& $\delta_o/D_c = [\{(1+\nu)\gamma_0+1-\nu\}p_c - 2\gamma_0 p_o]/E(\gamma_0-1) + \alpha_o\,\delta T_o$ – outer
Inserting these into compatibility equation $\delta_o - \delta_i = \Delta$, the RHS of (4) assumes the form.

$$[\,..\,]p_c = \Delta/D_c + \alpha_i\,\delta T_i - \alpha_o\,\delta T_o + 2\left[\frac{p_i}{E(\gamma_0-1)}\Big|_i + \frac{\gamma_0 p_o}{E(\gamma_0-1)}\Big|_o\right].$$

– and if the materials are the same, as in Ex 11 above:

$$\left[\frac{\gamma_0+1}{\gamma_0-1}\Big|_i + \frac{\gamma_0+1}{\gamma_0-1}\Big|_o\right]p_c = E[\Delta/D_c + \alpha(\delta T_i - \delta T_o)] + 2\left[\frac{p_i}{(\gamma_0-1)}\Big|_i + \frac{\gamma_0 p_o}{(\gamma_0-1)}\Big|_o\right]$$

Inserting values $\gamma_0|_i = \left(\frac{110}{100}\right)^2$ $\gamma_0|_o \Rightarrow \infty$ $\Delta = -0.05$ mm $p_i = 1$ MPa
$D_c = 110$ mm $E = 207\times10^3$ MPa $p_o = 0$ gives $p_c = 6.2$ MPa

13 (a) Estimate the contact pressure from (xii)
$p_c = 2T/\pi f D_c^2 L = 2\times7.3\times10^6/\pi/3\times130^2\times125 = 6.6$ MPa
Determine required Δ from (4) with $p_i = p_o = 0$
$\left[\frac{\gamma_0+1}{\gamma_0-1}\Big|inner + \frac{\gamma_0+1}{\gamma_0-1}\Big|outer\right]p_c = E\Delta/D_c$ – for same material
γ_0 inner $= (130/100)^2 = 1.69$ γ_0 outer $= (155/130)^2 = 1.422$
$\therefore \Delta = 9.64\times6.6\times130/207\times10^3 = \underline{0.04}$ mm

(b) Determine equivalent stress at different d from – first derive constants $\bar\sigma$, $\tilde\sigma$, $\tilde\tau$ $\boxed{\sigma_e^2 = \bar\sigma^2 + 3[(\tilde\sigma/\rho)^2 + \tilde\tau^2\rho]}$

INNER. $p_i = 0$ $p_o = 6.6$ MPa $\gamma_0 = 1.69$ $D_i = 100$ mm.
from (2) $\bar\sigma = \tilde\sigma = -16.2$ MPa
from (xvii) $\tilde\tau = 16\times7.3\times10^6/\pi\times10^6(1.69^2-1) = 20.0$ MPa
from (xviii)

d	100	105	110	115	120	125	130	mm
σ_E	47.4	47.3	47.5	48.0	48.7	49.6	50.7	MPa

OUTER $p_i = 6.6$ MPa $p_o = 0$ $\gamma_0 = 1.422$ $D_i = 130$ mm
from (2) $\bar\sigma = 15.7$ $\tilde\sigma = 22.3$ MPa
from (xvii) $\tilde\tau = 16\times7.3\times10^6/\pi\times130^3(1.422^2-1) = 16.6$ MPa
from (xviii)

d	130	135	140	145	150	155	mm
σ_E	50.5	49.1	48.0	47.2	46.7	46.4	MPa

It is evident that torque is dominant for the inner tube here, pressure for the outer. This is an efficient design as the stress level is practically uniform across both shafts. But see notes regarding approximations assumed.

(c) The maximum equivalent stress in the assembly is $\sigma^* = 50.7$ MPa.
It is necessary to check also the individual shafts at some remove from the coupling as it is possible that the contact pressure may compensate torsional shear.
Inner $\hat\tau = T\hat r/J = 7.3\times10^6\times130/2\ \frac{\pi}{32}(130^4-100^4) = 26.0$ MPa
Outer $\hat\tau = 7.3\times10^6\times155/2\ \frac{\pi}{32}(155^4-130^4) = 19.8$ MPa
So $\sigma^* = 50.7$ MPa overall – $n = 250/50.7 = \underline{4.9}$