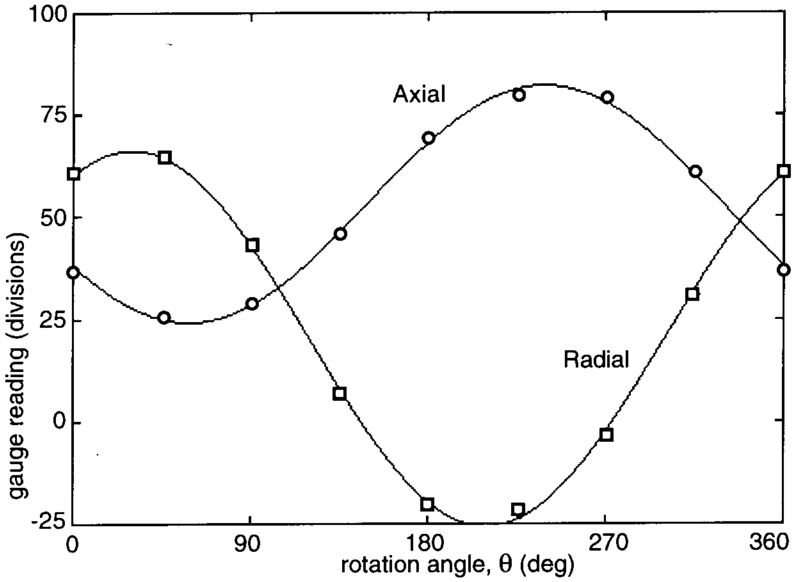
9



We require only the moments in vertical & horizontal planes ($\theta = 0, 90, 180, 270°$) but since relatively large random errors are usually experienced, it's better to measure moments at 45° intervals and then best-fit a sinusoid thru' them.

The given data are plotted here along with the regressions:

(i) $\epsilon_r = 20.3 + 45.9 \cos(\theta - 29.6°)$ as it happens here,
$\epsilon_a = 53.3 + 28.8 \cos(\theta + 121.1°)$ errors are small.

[As an aside, the regression constants were obtained by minimising the sum of errors, δ, squared in the approximation:

$$\delta = \epsilon_0 \cos(\theta - \theta_0) - \epsilon$$
$$= a_s \sin\theta + a_c \cos\theta - \epsilon$$

where $a_s \equiv \epsilon_0 \sin\theta_0$, $a_c \equiv \epsilon_0 \cos\theta_0$ and ϵ is the experimental value at θ.

The sum of errors S is then:

$$S = a_s^2 \Sigma \sin^2\theta + 2 a_s a_c \Sigma \sin\theta\cos\theta - 2 a_s \Sigma \epsilon \sin\theta + a_c^2 \Sigma \cos^2\theta - 2 a_c \Sigma \epsilon \cos\theta + \Sigma \epsilon^2$$

Setting $\partial S/\partial a_s = \partial S/\partial a_c = 0$ to minimise S:

(ii) $$a_s \Sigma \sin^2\theta + a_c \Sigma \sin\theta\cos\theta = \Sigma \epsilon \sin\theta$$
$$a_s \Sigma \sin\theta\cos\theta + a_c \Sigma \cos^2\theta = \Sigma \epsilon \cos\theta$$

whose solution yields a_s, a_c hence ϵ_0, θ_0]

Returning to the problem, from (i)

@ $\theta = 0°$	$\epsilon_r = 60.2$	$\epsilon_a = 38.4$	} i.e. in vertical plane
$= 180°$	$= -19.6$	$= 68.2$	
$= 90°$	$\epsilon_r = 43.0$	$\epsilon_a = 28.6$	} horizontal plane.
$= 270°$	$= -2.4$	$= 78.0$	

Note that these are not much different from given values – but we were not to know that errors were small until regression had been carried out.