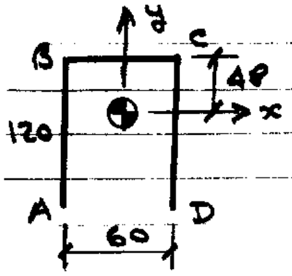
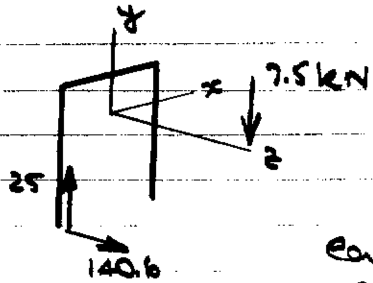
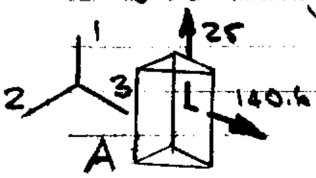
4





Out-of-plane bending.
– geometric properties
From tables (noting axes interchange)
$b = 120 \quad d = 60 \quad L = 300$
Centroid as shown, at $b^2/L = 48$ mm from "web."

$I_{xx} = b^3(b+2d)/3L = 460.8 \times 10^3 \text{ mm}^3$
Note that $I_{xy} = 0$ (symmetric) & I_{yy} not needed since force in y-z plane & no bending about y-axis.
Maximum intensity at A & D since y is max. for these
Primary $q_P = F/L = 7.5e3/300 = 25$ N/mm
Secondary $q_S = My/I_{xx} = (7.5 \times 120)e3 \times (120-48)/460.8e3 = 140.6$
– hence components at A as illustrated.



The primary 25 N/mm is q_1 loading, the secondary q_3 – by inspection
Hence $q_E = \sqrt{(1.5 \times 25^2 + 140.6^2)} = 143.9$ N/mm
$\therefore \sigma_E = 2q_E/w = 2 \times 143.9/6 = \underline{\underline{48.0}}$ MPa.

5 $\quad \mathbf{b \times r} = \begin{vmatrix} i & j & k \\ b_x & b_y & b_z \\ x & y & z \end{vmatrix} = \begin{bmatrix} b_y z - b_z y \\ b_z x - b_x z \\ b_x y - b_y x \end{bmatrix}$

$\mathbf{r \times (b \times r)} = \begin{vmatrix} i & j & k \\ x & y & z \\ (b_y z - b_z y) & (b_z x - b_x z) & (b_x y - b_y x) \end{vmatrix} = \begin{bmatrix} b_x(y^2+z^2) - b_y xy - b_z xz \\ -b_x xy + b_y(x^2+z^2) - b_z yz \\ -b_x xz - b_y yz + b_z(x^2+y^2) \end{bmatrix}$

Integrating the terms of this vector one-by-one:
$\int_L (y^2+z^2)\, dL = J_{xx}, \quad \int_L xy\, dL = I_{xy}$ etc etc.
and so the integral becomes:-

$\int_L \mathbf{r \times (b \times r)}\, dL = \begin{bmatrix} b_x J_{xx} - b_y I_{xy} - b_z I_{xz} \\ -b_x I_{xy} + b_y J_{yy} - b_z I_{yz} \\ -b_x I_{xz} - b_y I_{yz} + b_z J_{zz} \end{bmatrix} = \begin{bmatrix} J_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & J_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & J_{zz} \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$

The proof of the last step follows from matrix multiplication
i.e. $\int_L \mathbf{r \times (b \times r)}\, dL = \mathbf{I\, b}$ hence eq (4)

6 The line may be expressed parametrically as:-
$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \equiv \lambda$
where λ is the fractional distance along the line – i.e. $\lambda = 0$ at point 1, $\lambda = 1$ at point 2.

So: $x = x_1 + \lambda . \Delta x \; ; \; \Delta x = x_2 - x_1$
$y = y_1 + \lambda . \Delta y \; ; \; \Delta y = y_2 - y_1$
$z = z_1 + \lambda . \Delta z \; ; \; \Delta z = z_2 - z_1$
and total length $L = \sqrt{[\Delta^2 x + \Delta^2 y + \Delta^2 z]}$

From the integral definitions, with $dl = L . d\lambda$:
$J_{xx} = L \int_0^1 [(y_1 + \lambda . \Delta y)^2 + (z_1 + \lambda . \Delta z)^2]\, d\lambda$
$I_{xy} = L \int_0^1 [x_1 + \lambda . \Delta x][y_1 + \lambda . \Delta y]\, d\lambda$
lead to the quoted results.